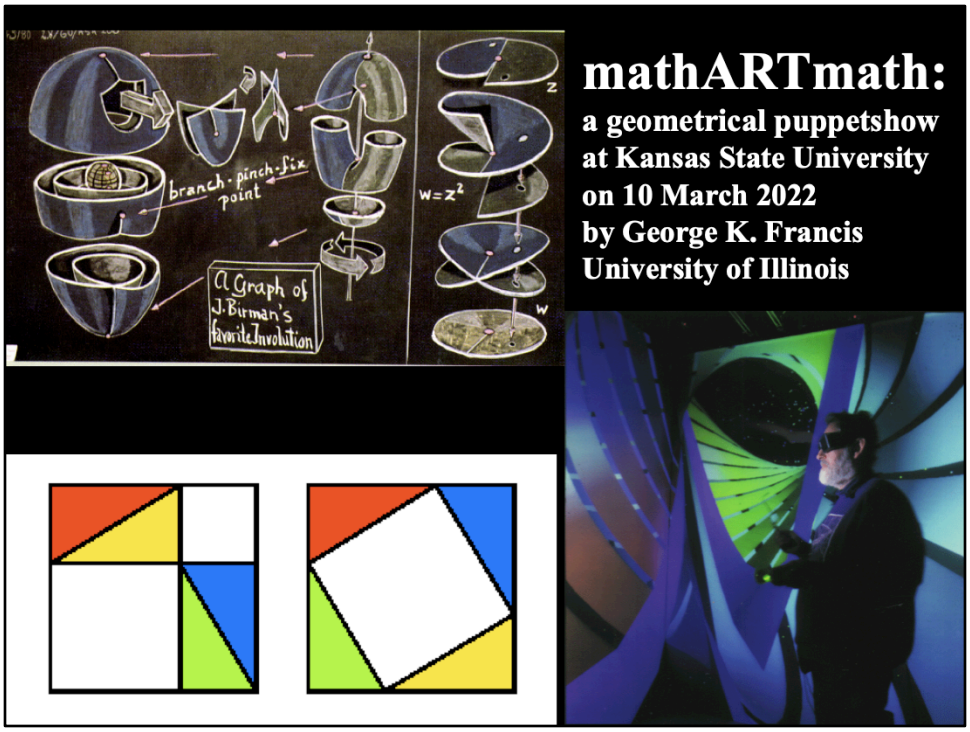
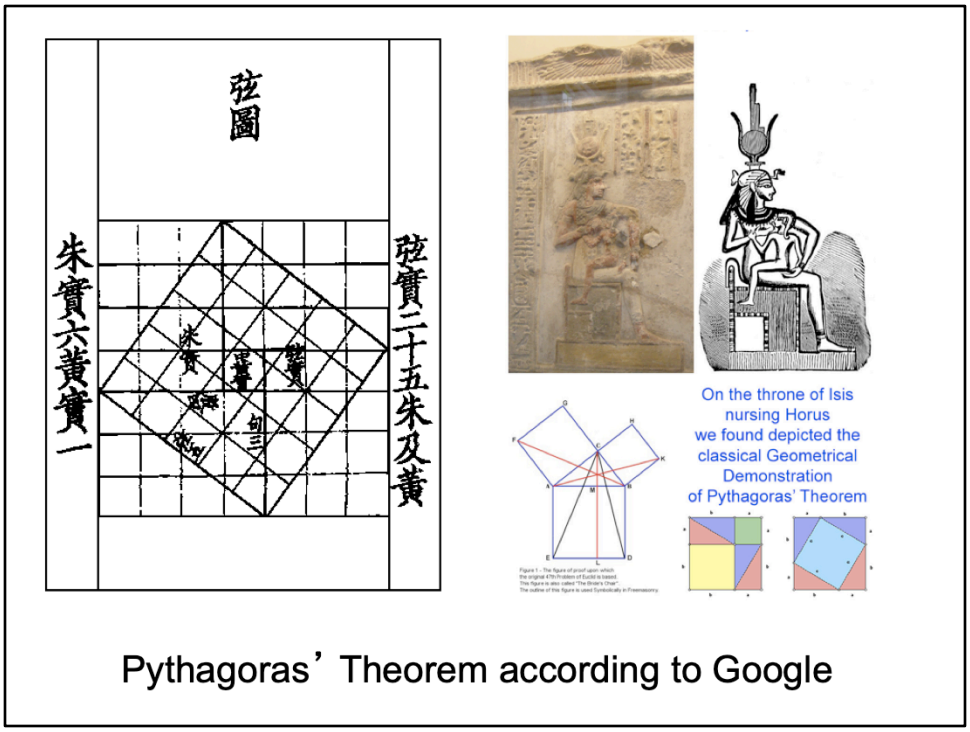


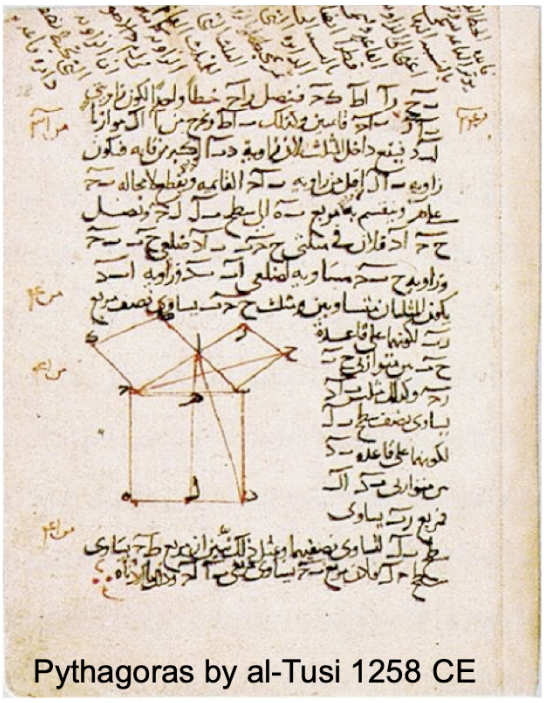

Commissioned figure drawn on a blackboard. The complex equivalent of the 9th grade parabola.
Surely the best know picture proof. Can you recognize the theorem? But why is it NOT rigorous?

Let's start with the two things eveybody remembers from HS, y=x^2 but over C and the Pythagorian Theorem. The latter is easier.




Pythagoras' Theorem according to Google

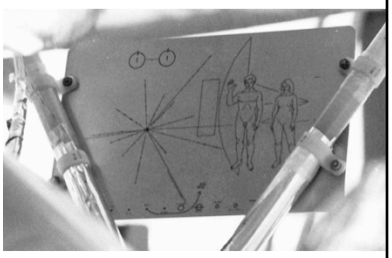
The ancient Chinese clearly knew the theorem. But apparently, the Egyptians didn't .... or the author of what Google found did not.
But, did the Chinese ancients know a proof? The so-called Chinese proof, lower right, is wrong, as most teachers tell it.
Euclid's proof, comparing the squares on the sides to the square under the triangular gable is a proof.



Pythagoras by al-Tusi 1258 CE

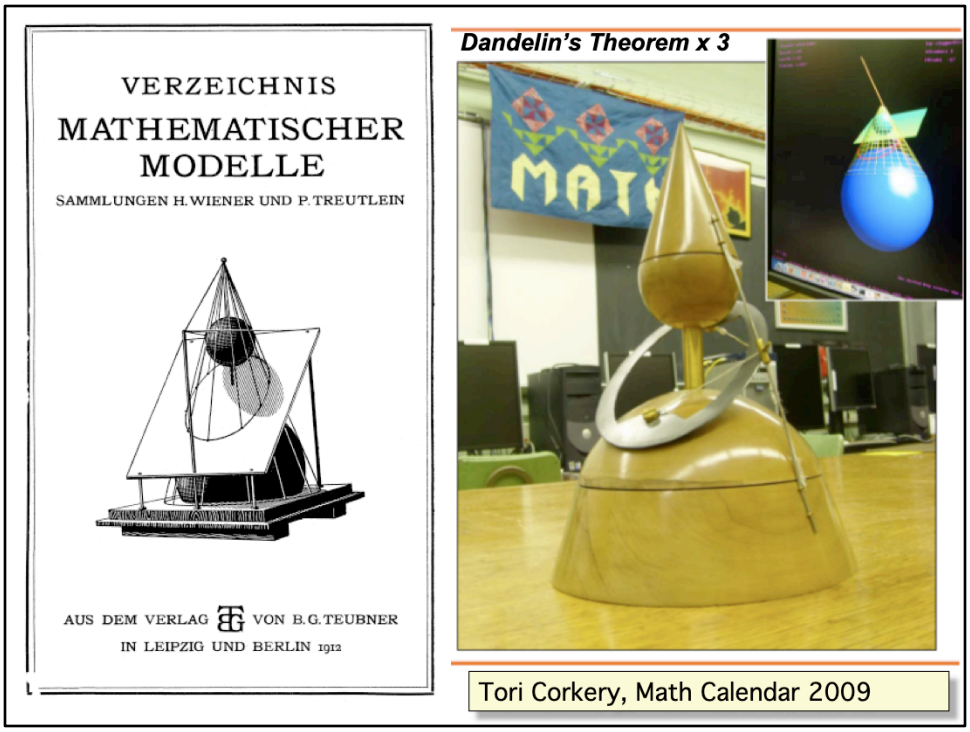
The picture for Euclid's proof was chosen for the plaque on Pioneer 10 not so much to prove that we knew the theorem, but that we knew how to prove it

Yuri Rainich, 1955



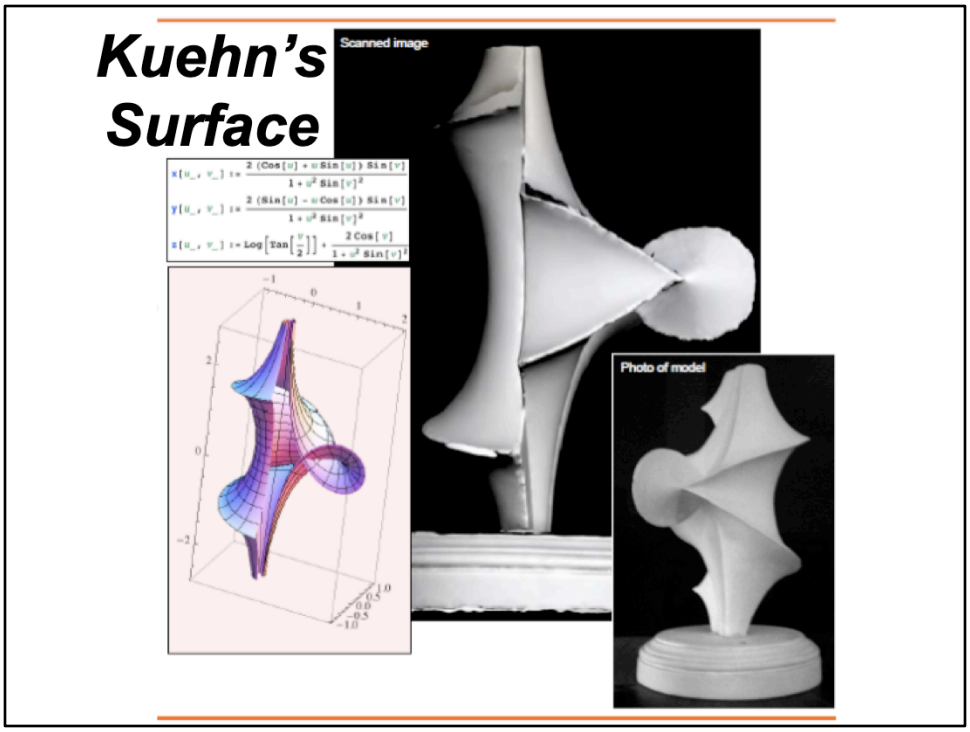
http:// www.homodiscens.com
Euclid's proof of the Pythagorean theorem was rendered into Arabic in AD 1258 by the Persian mathematician al-Tusi.
Pioneer 10 and 11, 1972 NASA

Not so easy to understand with al-Tusi's lack of artistic skill.
Alas, Pioneer passed out of our solar system wihout an engraving of Euclid's picture.




Dandelin's Theorem. Spheres tangent to cone and plane touch the foci of the conic section.
Altgeld model collection. Klein at Erlangen, Munich TH, Leipzig, Goettingen. (with Brill at MTH)
Conic sections teacher's workshop at NCSA 1992 w. Chris Hartman and Francois Apery.

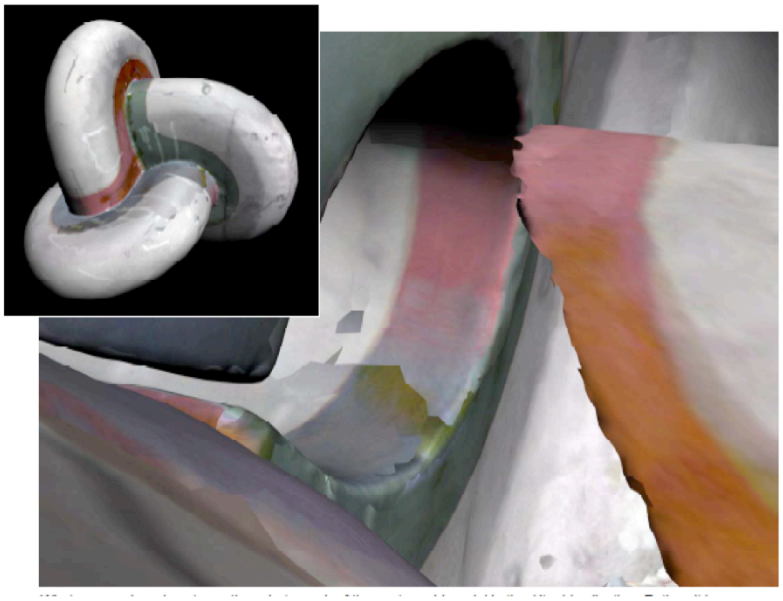
Contnuing our whirlwind tour through history of mathARTmath,
there are familiar models, many designed by Felix Klein's undergraduate students in the four German universities he taught.



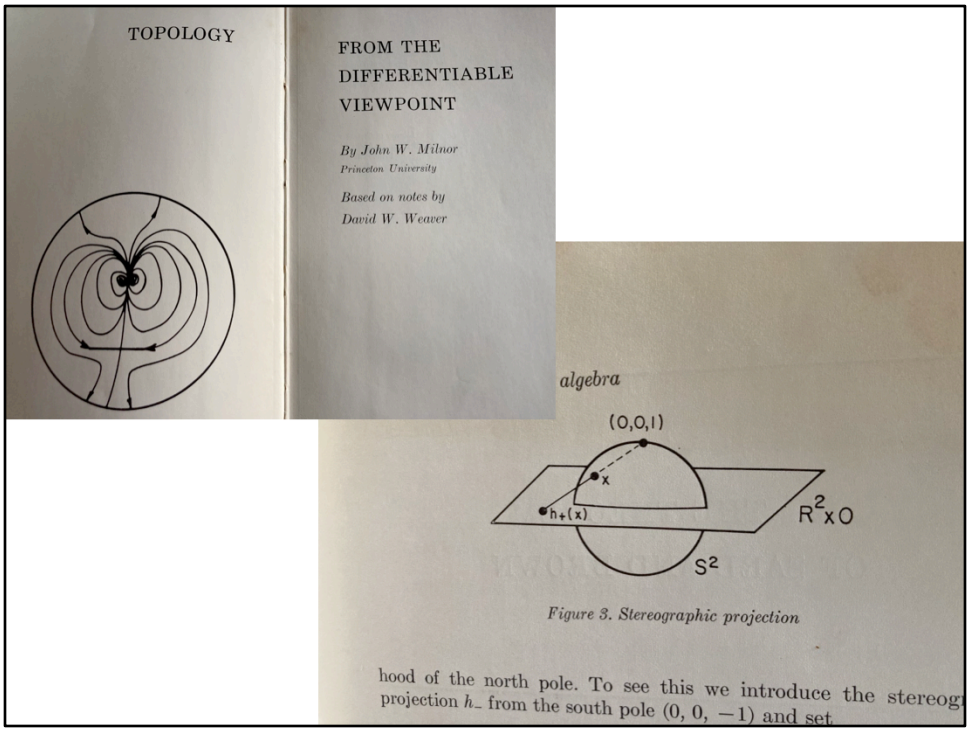

Kuehn's surface. Model in case near Math Office.
Scanned by Abby Watts for CalculArt http://www.isl.uiuc.edu/canvas/dennos/.
Bruce Carpenter's Mathematica code and figure.

This tableau show

Inside view of Abby Watt's scan of the plaster Boy's surface. Calculart 2008



In the Cube you can incongruously fly inside a solid plaster model.

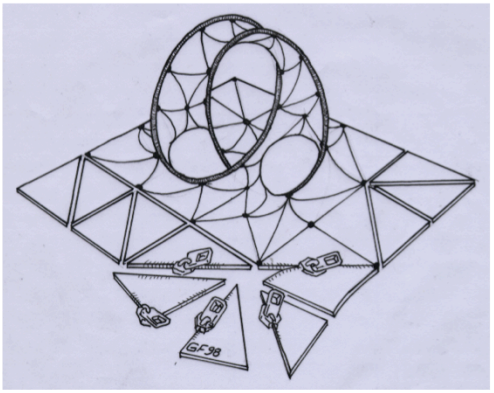



A typical illustrations that would have benefitted from artistic skills .... of the author. Milnor undoubtedly sketched the image, and the no doubt artistically trained copied all
the errors:  Elliptical equator, apparent discontinuity due to the perspective error, north pole at the top of the head
But John Milnor displays a delightful sense of humor in his frontice piecel. Humor is a rarity in math ... and art.

# Conway's ZIP Proof

Geore K. Francis and Jeffrey R. Weeks
Amer. Math. Monthly, 106 (May)1999

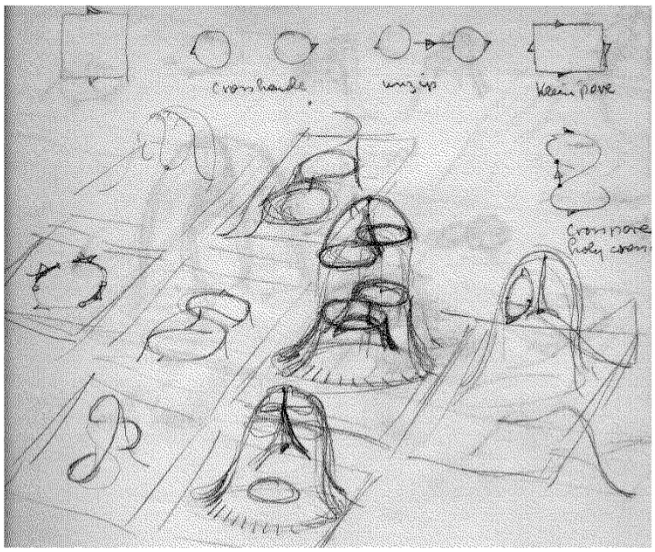
Surface = Sphere
with
Pores (holes)
Handles
Crosscaps
and
(unnecessary)
Crosshandles




Triangulated Seifert surface spanning a Hopf link

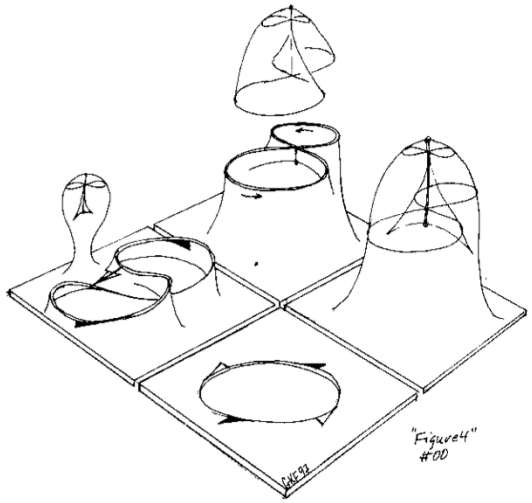
Our first tangential departure is the back story of the illustrations for this work.

## Paper napkin sketches



G.Francis and Jeff Weeks, “Conway’s ZIP Proof, Amer Math Monthly, 1999

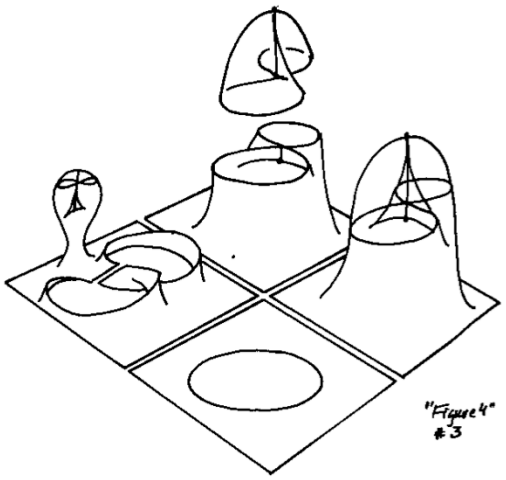
## Crosscap one




G.Francis and Jeff Weeks, "Conway's ZIP Proof, Amer Math Monthly, 1999

Public_html\zipproof\hopf.tiff

## Crosscap two




G.Francis and Jeff Weeks, "Conway's ZIP Proof, Amer Math Monthly, 1999

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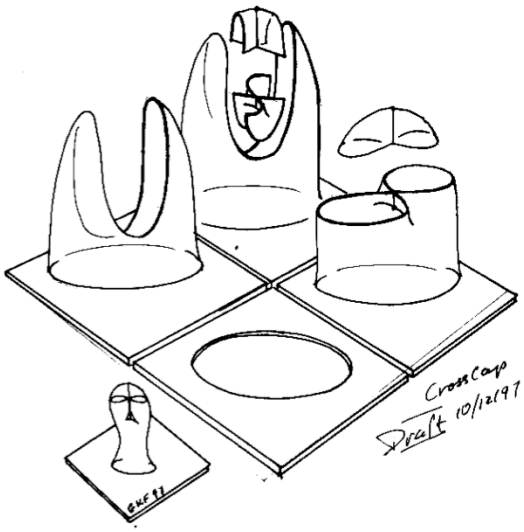
## Crosscap three




G.Francis and Jeff Weeks, "Conway's ZIP Proof, Amer Math Monthly, 1999

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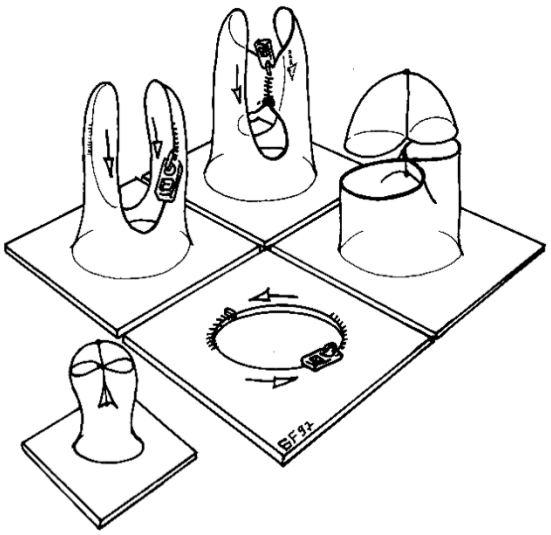
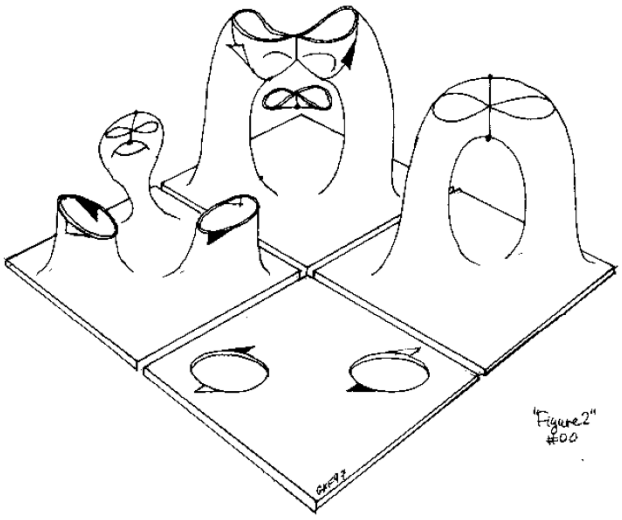
## Crosscap final



G.Francis and Jeff Weeks, "Conway's ZIP Proof, Amer Math Monthly, 1999

Public_html\zipproof\hopf.tiff

## Crosshandle one




G.Francis and Jeff Weeks, "Conway's ZIP Proof, Amer Math Monthly, 1999

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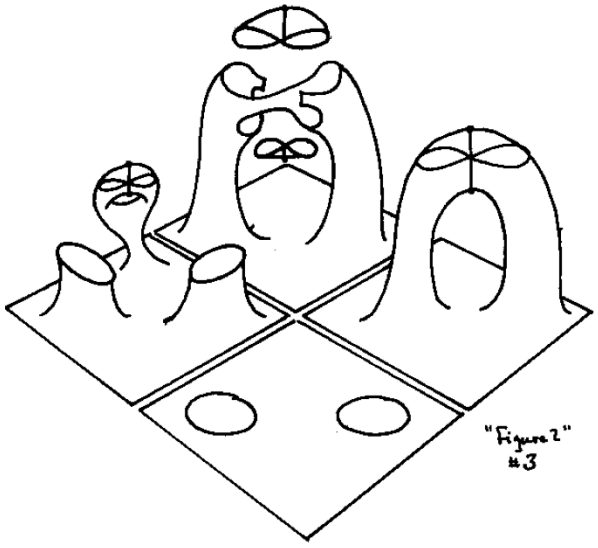
## Crosshandle two




G.Francis and Jeff Weeks, "Conway's ZIP Proof, Amer Math Monthly, 1999

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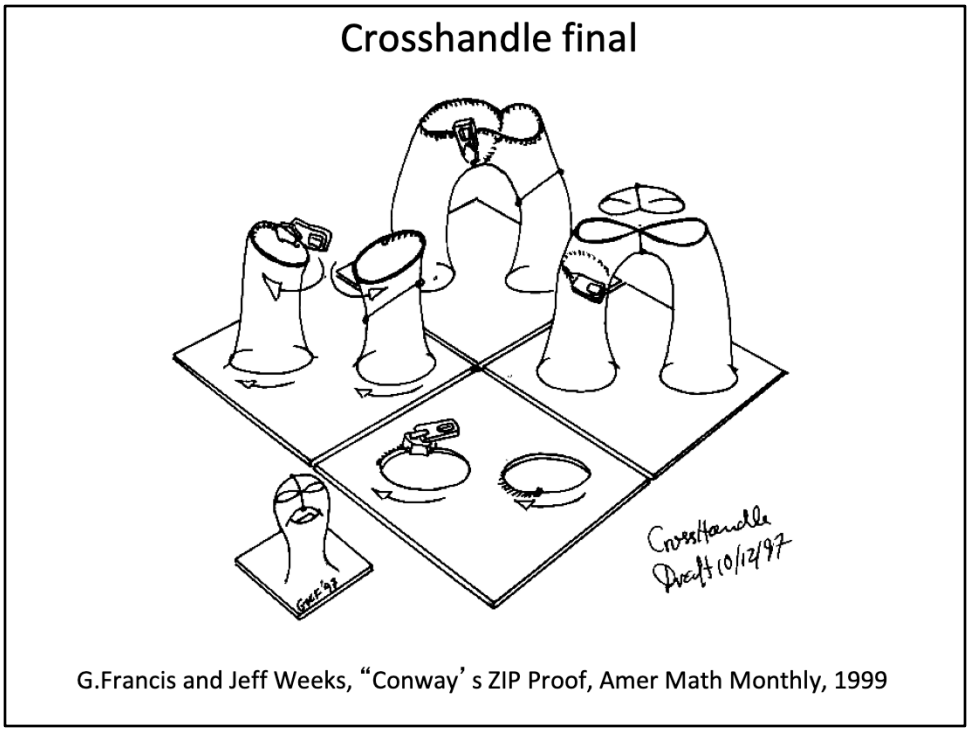


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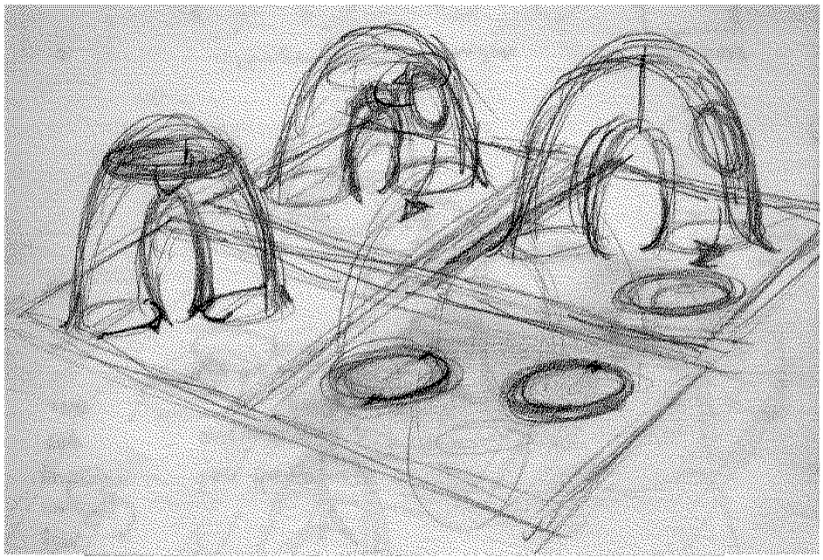
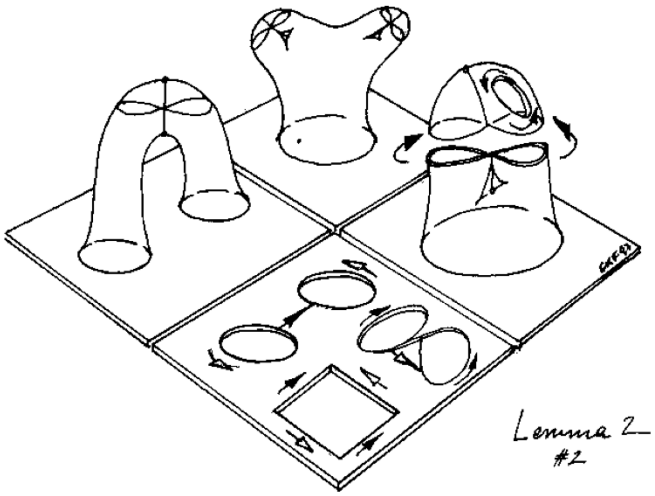
## Kleinpore sketch



G.Francis and Jeff Weeks, "Conway's ZIP Proof, Amer Math Monthly, 1999

Public_html\zipproof\hopf.tiff

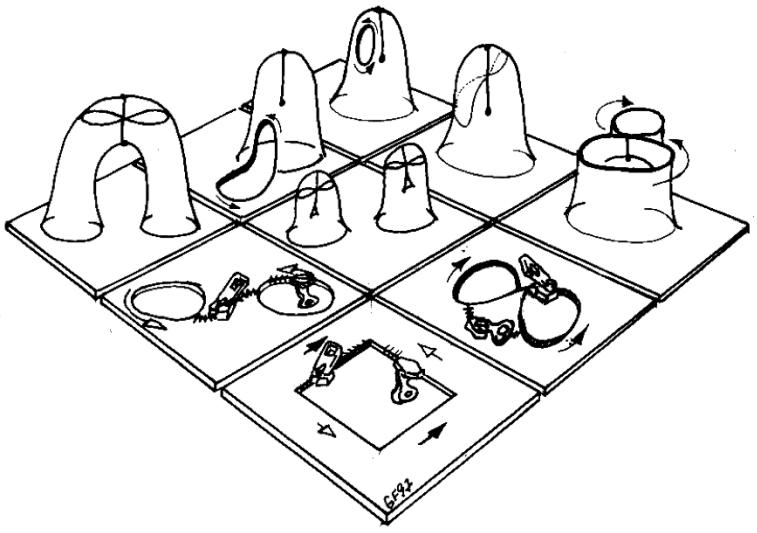
## Kleinpore, second draft




G.Francis and Jeff Weeks, "Conway's ZIP Proof, Amer Math Monthly, 1999

Public_html\zipproof\hopf.tiff

## Kleinpore: 1 xhandle = 2 xcaps



G.Francis and Jeff Weeks, "Conway's ZIP Proof, Amer Math Monthly, 1999

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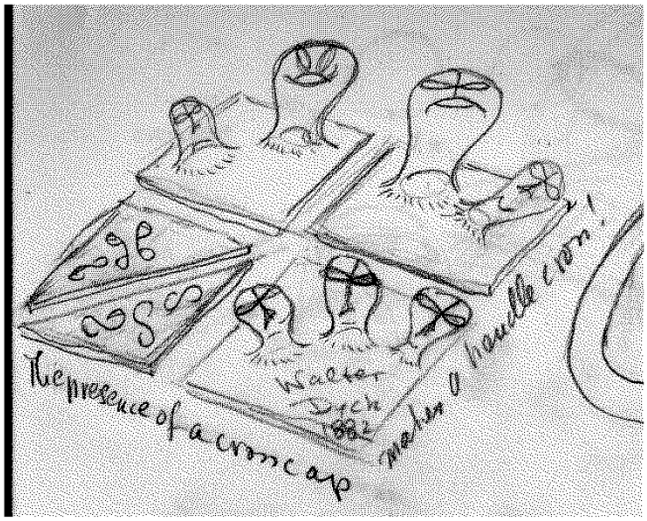
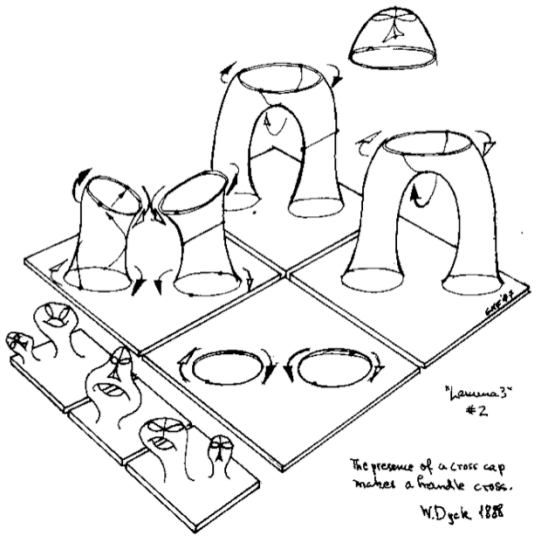
## Dyck's Theorem, first sketch




G.Francis and Jeff Weeks, "Conway's ZIP Proof, Amer Math Monthly, 1999

Public_html\zipproof\hopf.tiff

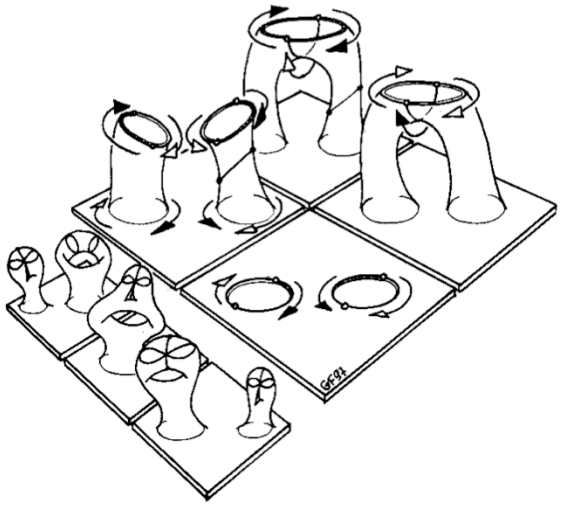
## Dyck's Theorem, second draft




G.Francis and Jeff Weeks, "Conway's ZIP Proof, Amer Math Monthly, 1999

Public_html\zipproof\hopf.tiff

## Dyck's Theorem: 3 xcaps = 1 xcap + 1 handle



G.Francis and Jeff Weeks, "Conway's ZIP Proof, Amer Math Monthly, 1999

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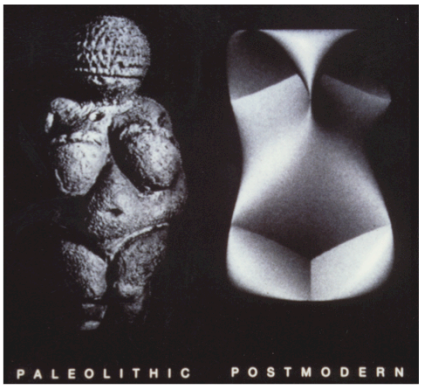
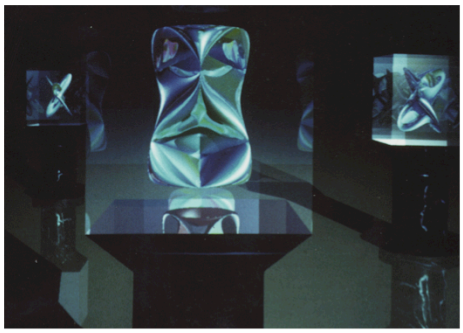
# Etruscan Venus

Cover Girls



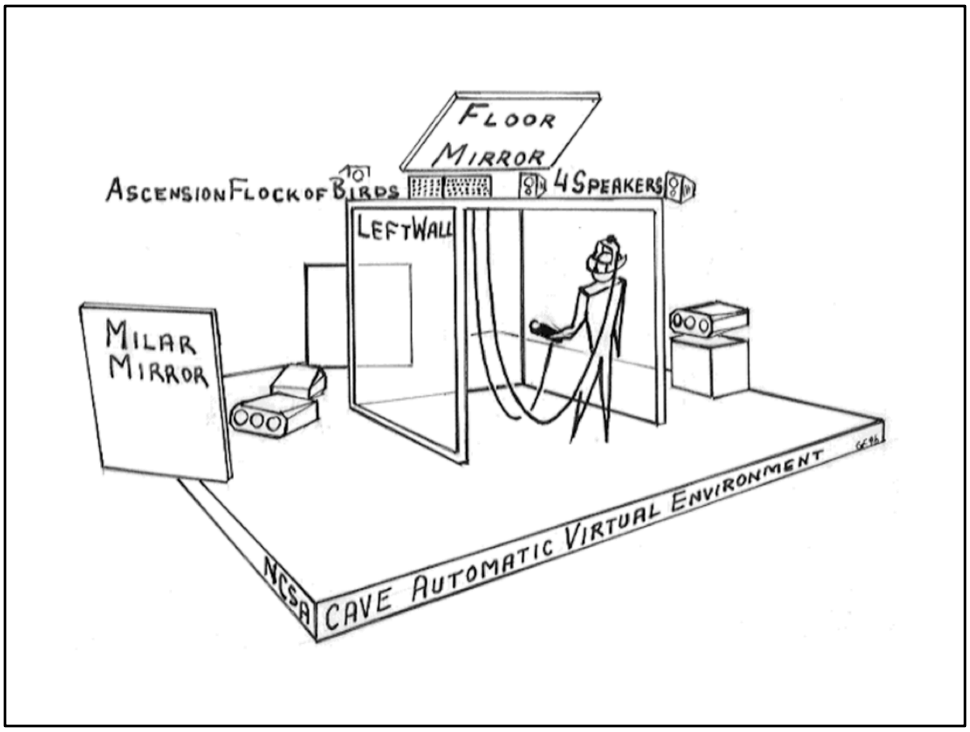
Crystalline Raytrace.

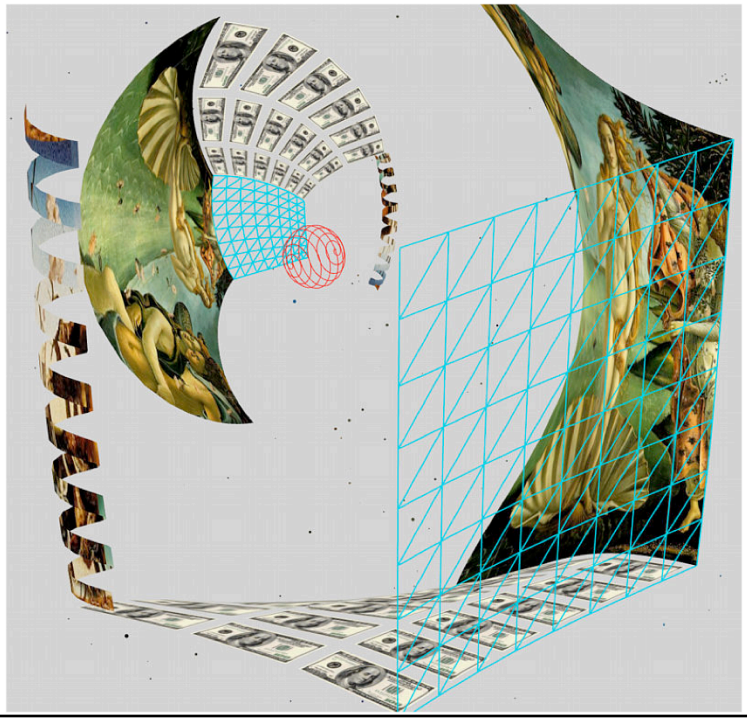
Donna Cox, George Francis, Ray Idaszak, NCSA ©1987

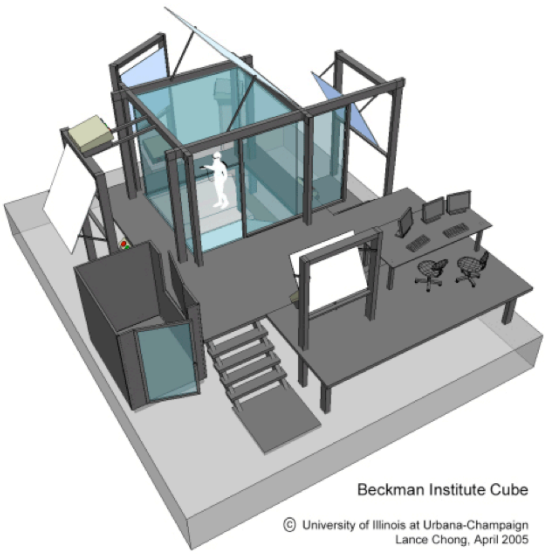
Nearly all of my work is a collaboration.

# *Apéry's Romboy Homotopy*

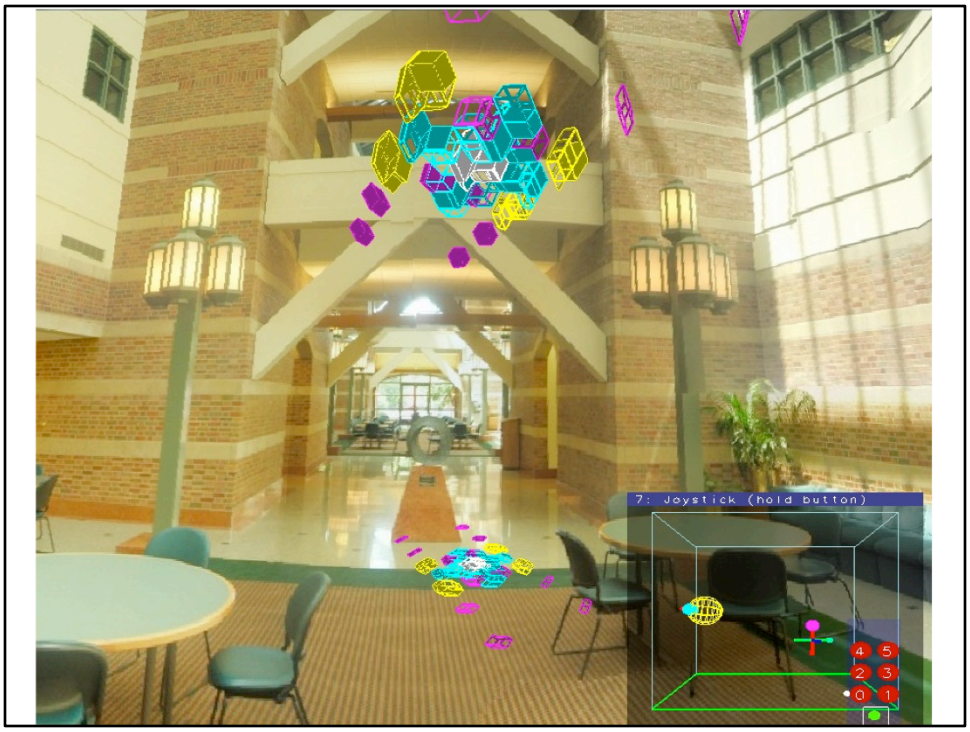
***A Real-time Interactive Computer Animation (RTICA)***

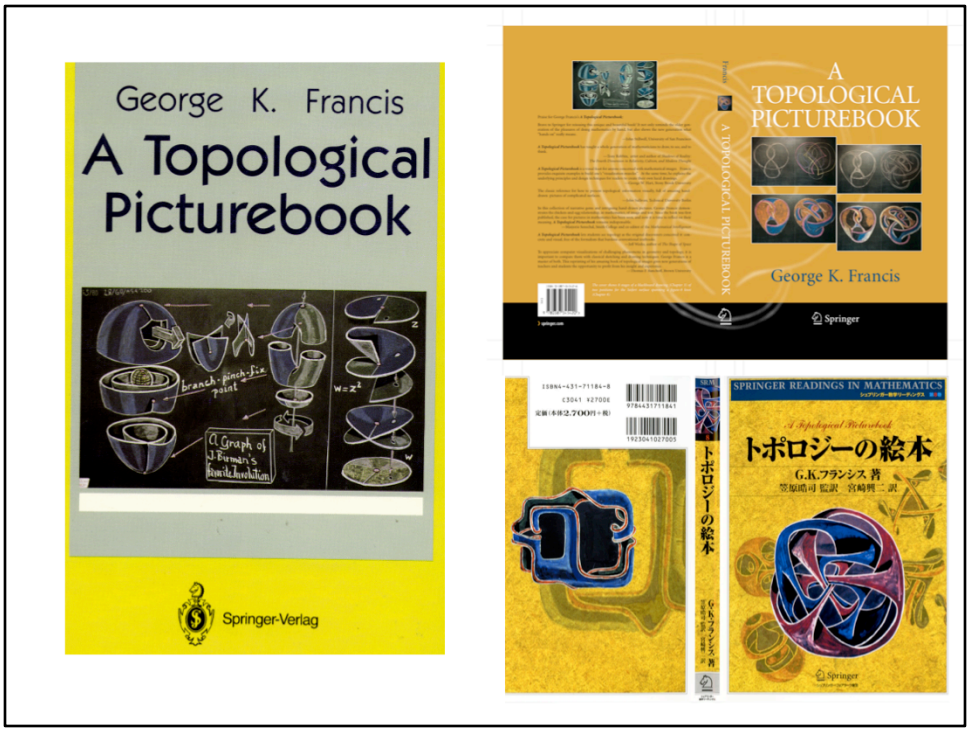

FLOOR
MIRROR
ASCENSION FLOCK OF BIRDS
4 SPEAKERS
LEFT WALL
MILAR
MIRROR
NCSA CAVE AUTOMATIC VIRTUAL ENVIRONMENT





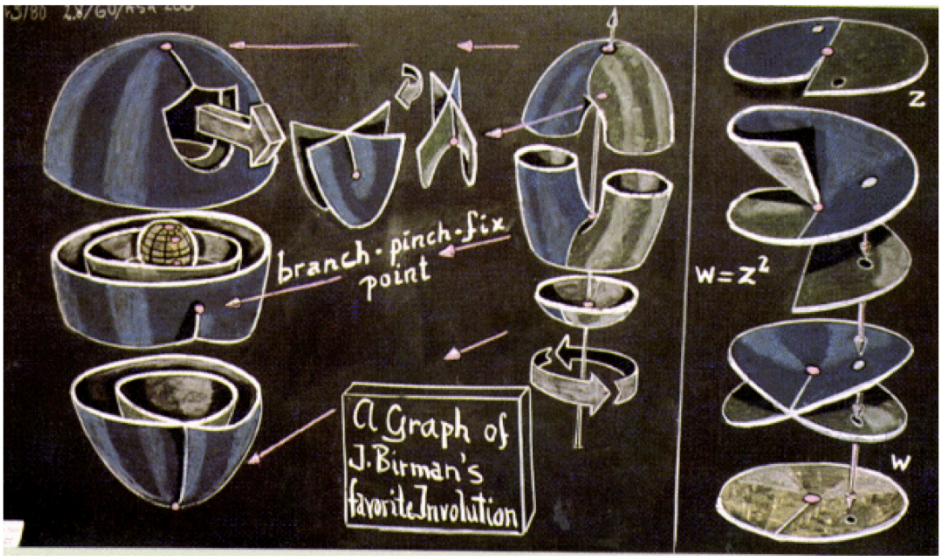
Beckman Institute Cube


7: Joystick (hold button)
4 5
2 3
0 1



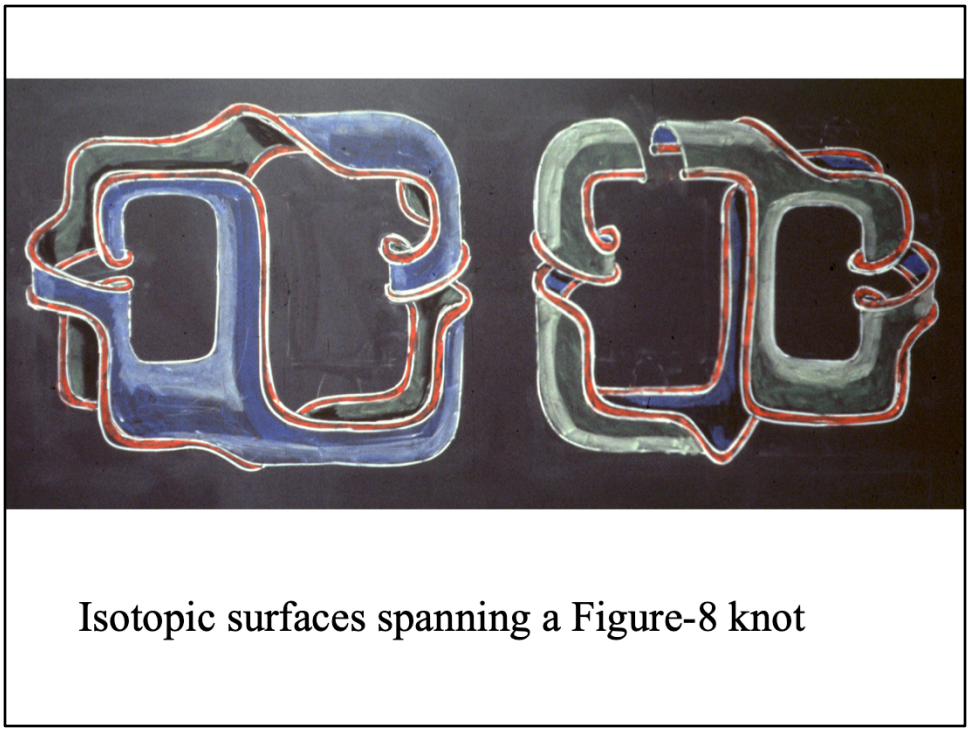

The editors for each edition or translation requested new cover art, except the Russian one, which has a black cover.

# Joan Birman's Favorite Involution




[get rid of the duplicated title] What squaring the complex numbers looks like as a Riemann surface.
The 4x branched double of the sphere by a torus by its involution of a 180º rotation about a skewer.


Unsymmetrical Triblague 1979

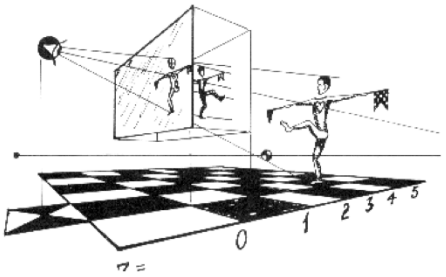


Isotopic surfaces spanning a Figure-8 knot

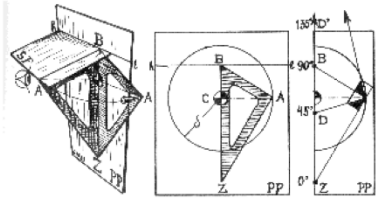
Thurston Seminar, eighties.
If you can see the isotpy through the interior of the double torus
Then you should study geometry/topology/group theory

# Linear Perspective

Renaissance
and
OpenGL

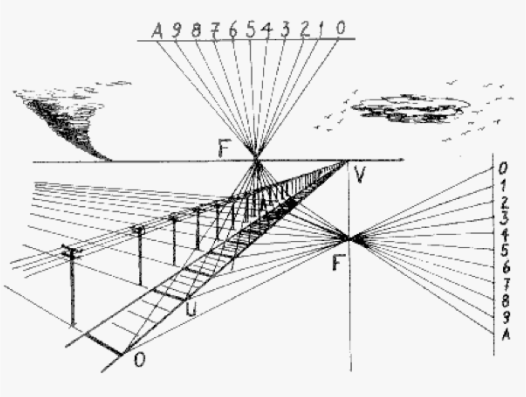



Horizon – Zenith
and the
Geometric Mean




from George K. Francis, "A Topological Picturebook", Springer-Verlag, 1987

# Prairie Horizon




Crossratios are the rulers in projective geometry

# Whitney Umbrella

original right and elaborated by Lun Yi Tsai below

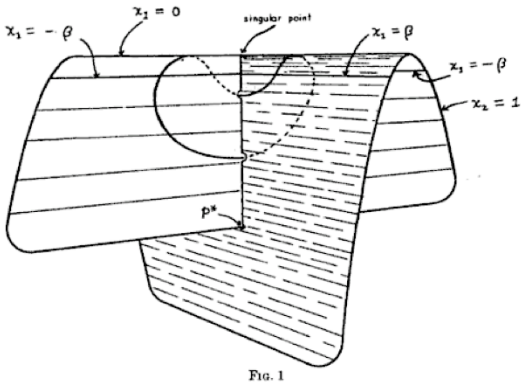
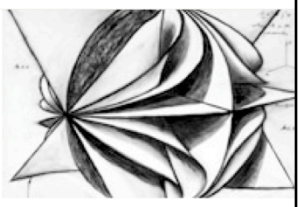
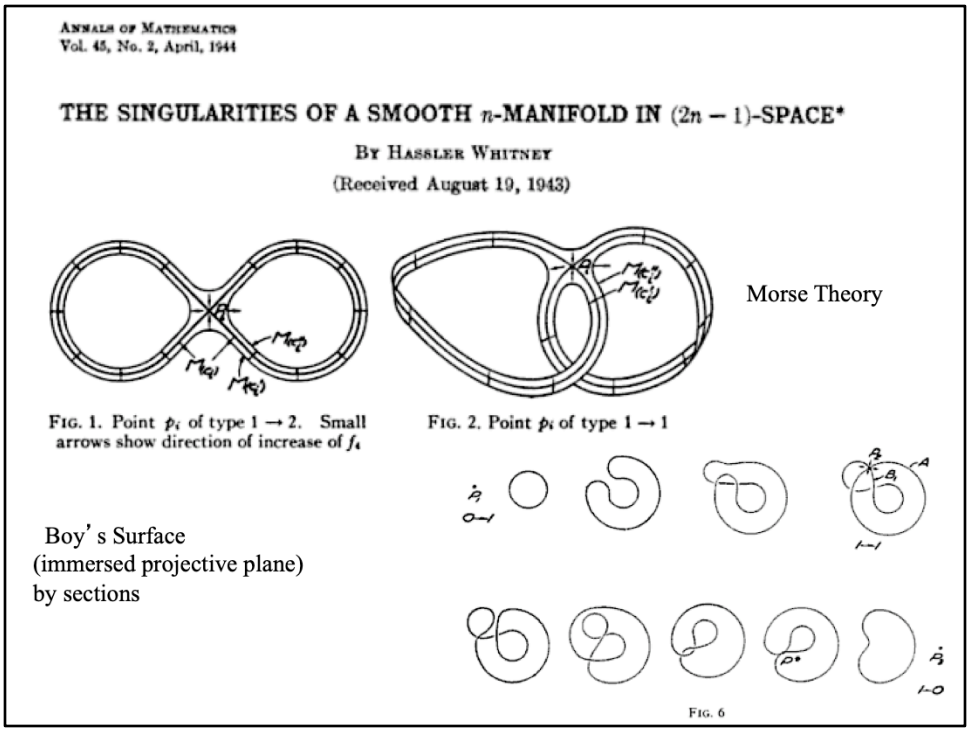


FIG. 1



*4. Whitney Family*
2008, charcoal and graphite on paper, 39 x 39 in

The Whitney Umbrella, a topological surface named for the American mathematician Hassler Whitney, can be thought of as a plane that is cut along a ray and glued back in such a way that it intersects with itself in three dimensions. This parameterization creates a stack of umbrellas sitting on their sides.



ANNALS OF MATHEMATICS
Vol. 45, No. 2, April, 1944

THE SINGULARITIES OF A SMOOTH $n$-MANIFOLD IN $(2n-1)$-SPACE*

BY HASSLER WHITNEY

(Received August 19, 1943)

Morse Theory

FIG. 1. Point $p_i$ of type $1 \to 2$. Small arrows show direction of increase of $f_4$

FIG. 2. Point $p_i$ of type $1 \to 1$

Boy's Surface
(immersed projective plane)
by sections

FIG. 6

Note the publication delay even then and even for the famous.
Morse theory for 3D sections in 4D
Boy's surface

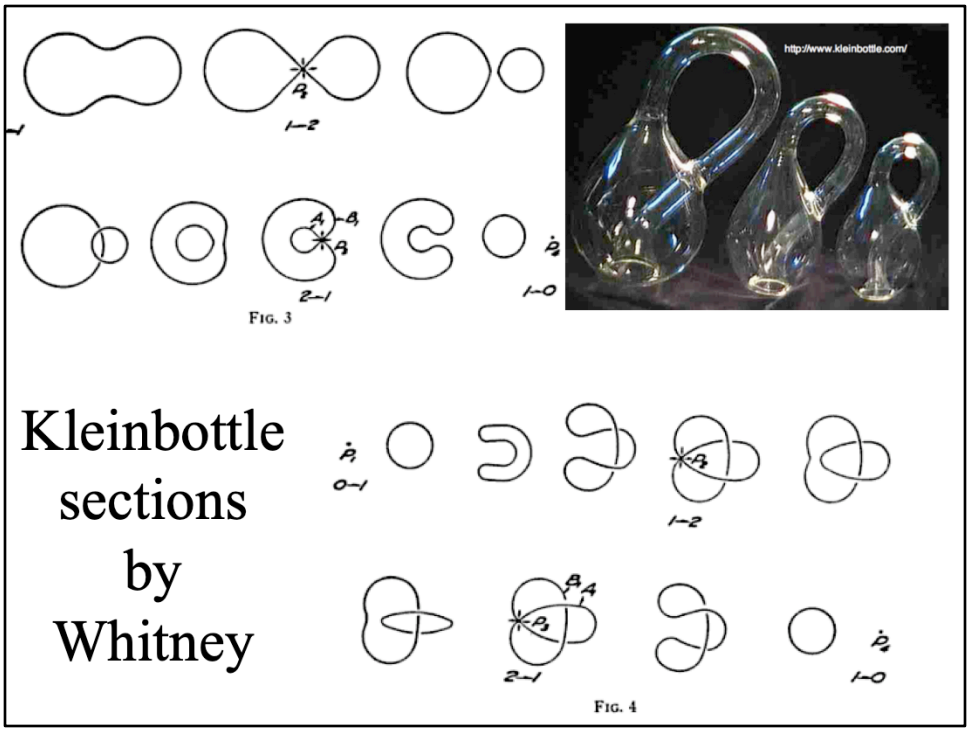
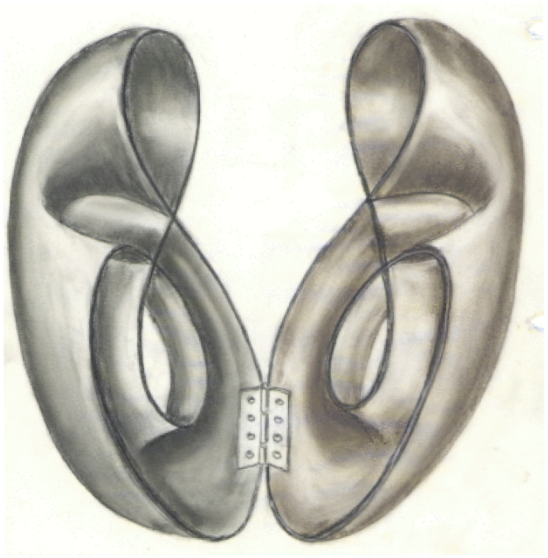



FIG. 3

FIG. 4

# Hinged Kleinbottle

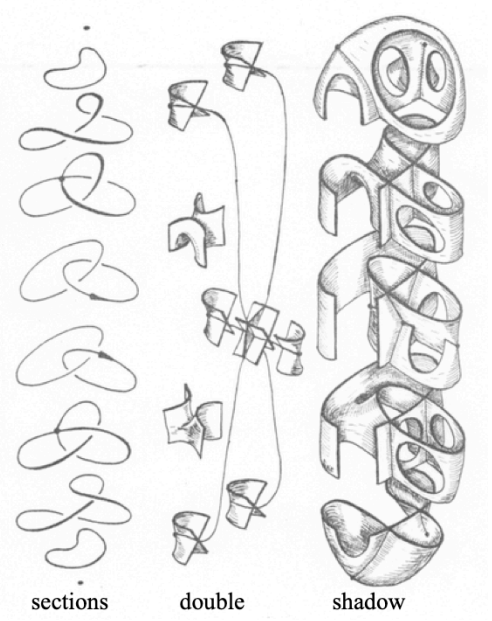


Pastel drawing of two immersed Möbius bands that form a Kleinbottle, 1983.

[either this here, or the two tried and abandonded media experiments]

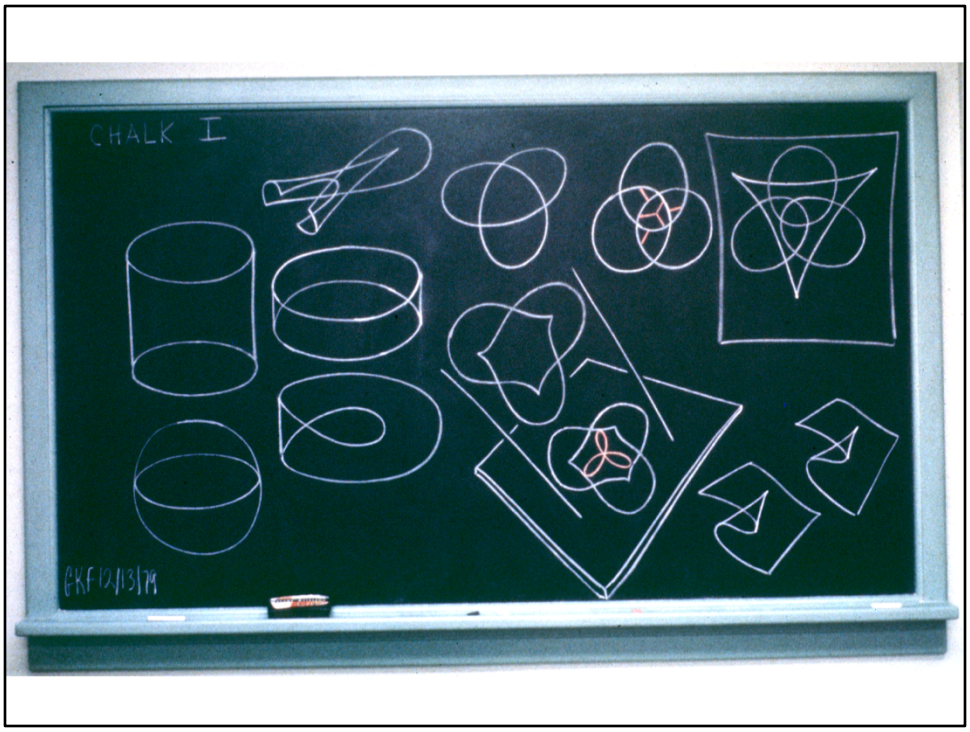
# Whitney Bottle

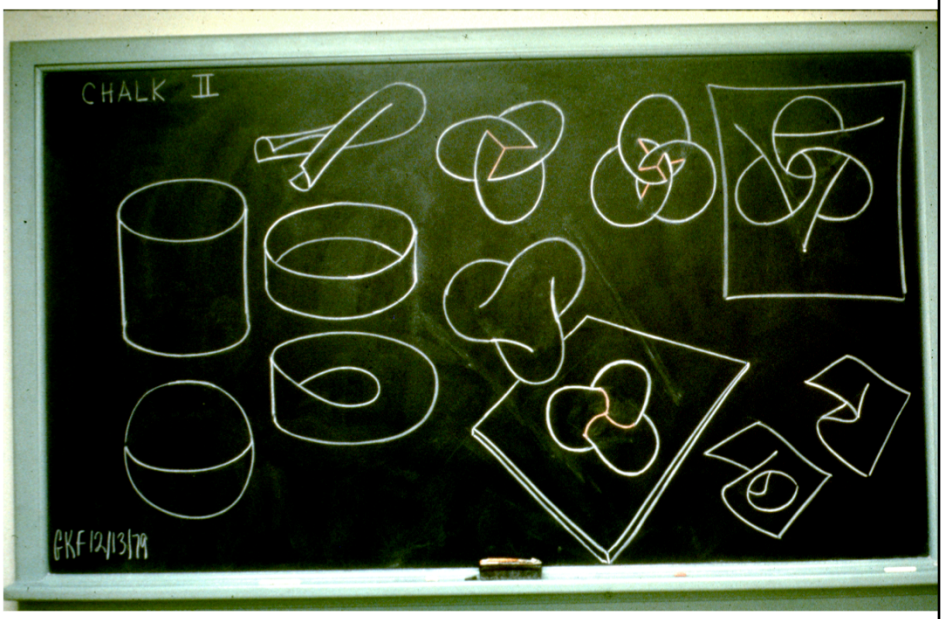
From
3-D sections
construct the
double locus,
& shadow of
a Kleinbottle
embedded in
4-Space

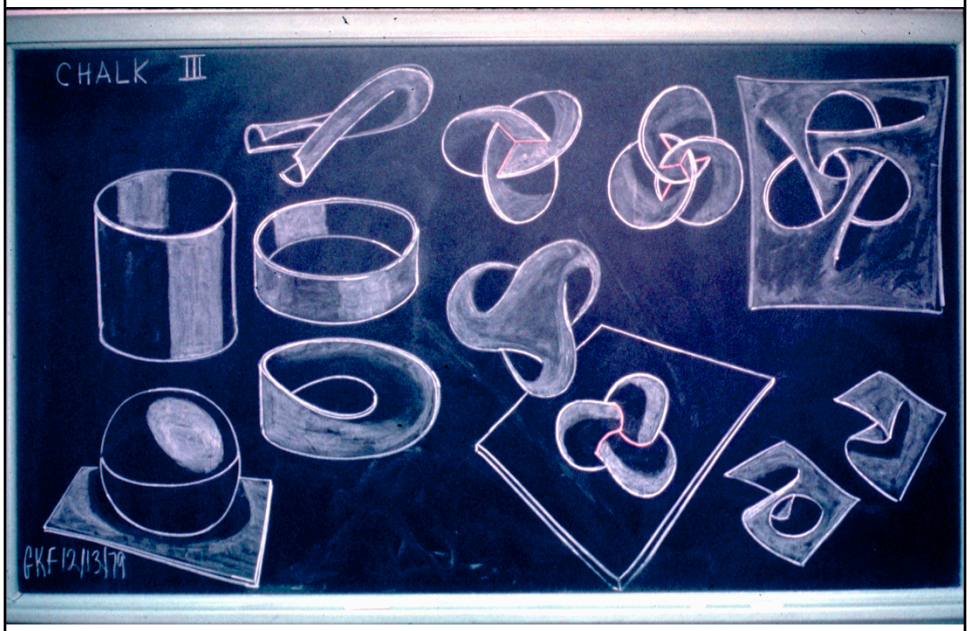


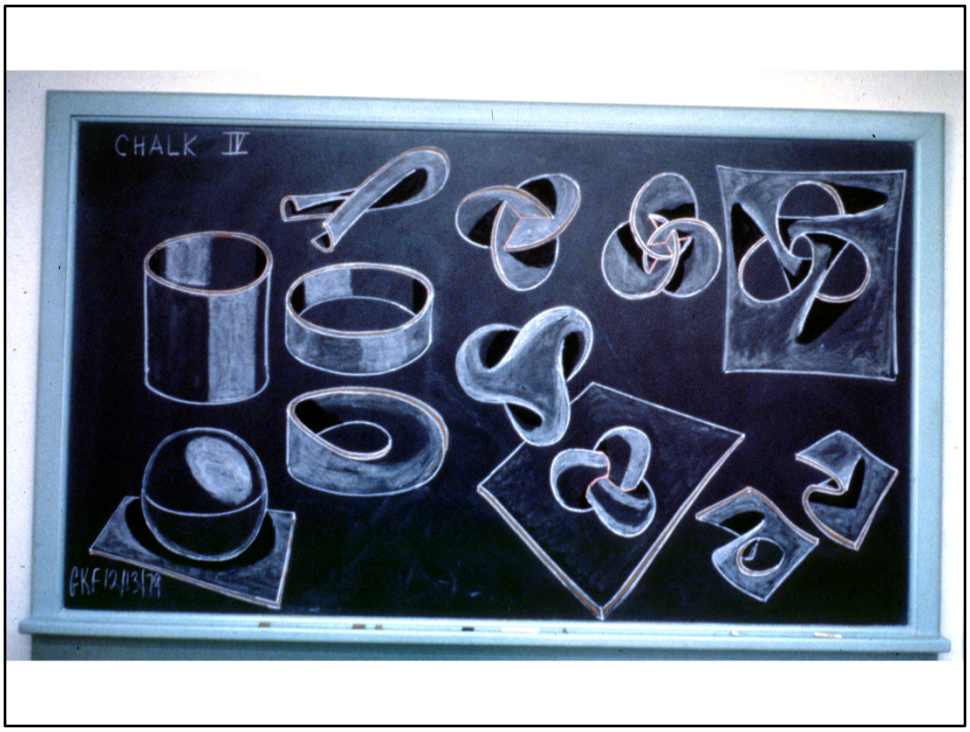

from George K. Francis, "A Topological Picturebook", Springer-Verlag, 1987


*Chalk on Blackboard*


CHALK I
GKF 12/13/79

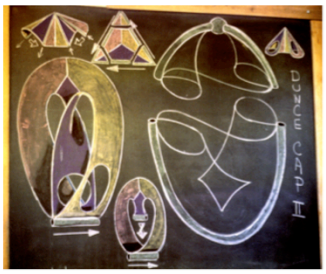

CHALK II
GKF 12/13/79


CHALK III
GKF 12/13/79


CHALK IV
GKF 12/13/74

# Blackboard Dunce Cap

line drawing



from George K. Francis, "A Topological Picturebook", Springer-Verlag, 1987

# Blackboard Dunce Cap

line drawing windows & shadows

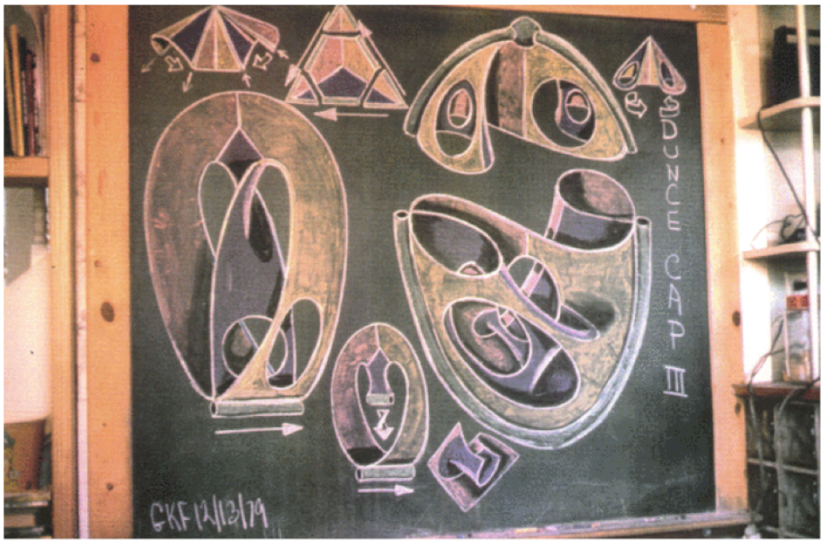
from George K. Francis, "A Topological Picturebook", Springer-Verlag, 1987

# Blackboard Dunce Cap

line drawing windows & shadows coloring

from George K. Francis, "A Topological Picturebook", Springer-Verlag, 1987

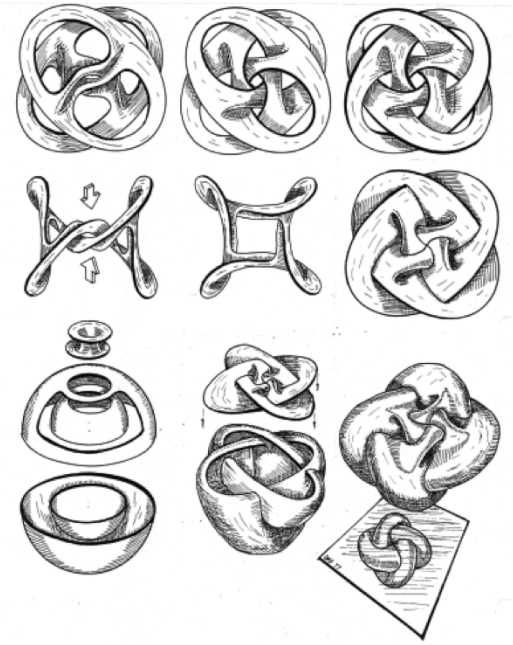
# Blackboard Dunce Cap




from George K. Francis, "A Topological Picturebook", Springer-Verlag, 1987

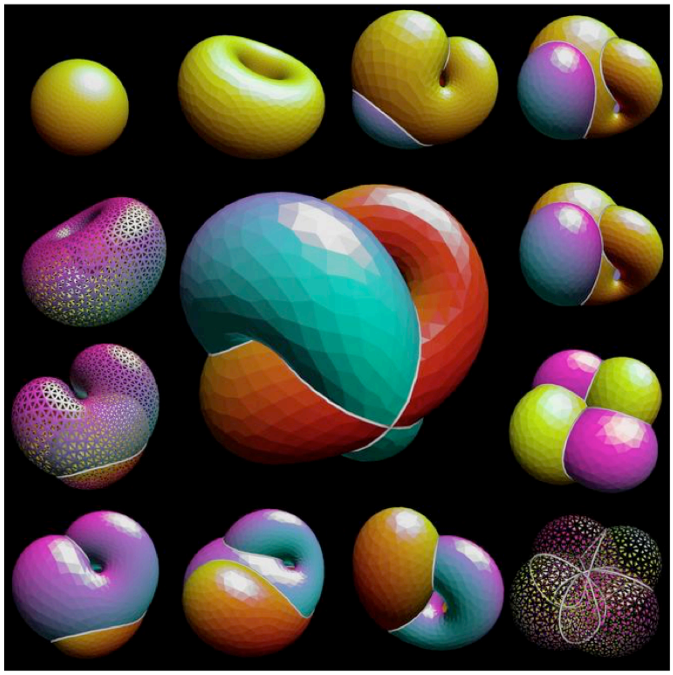
***John Dalbec's***
***Dunce Hat***
***RTICA***

Morin
Tableau
n=2



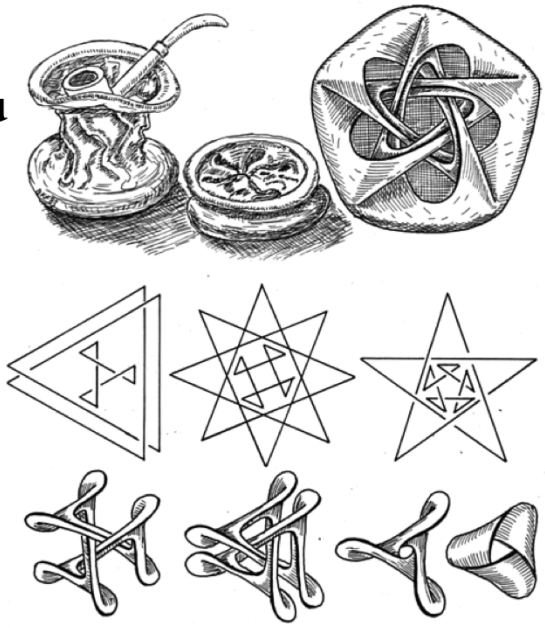
from George K. Francis, "A Topological Picturebook", Springer-Verlag, 1987

Morin
Montage
n=2



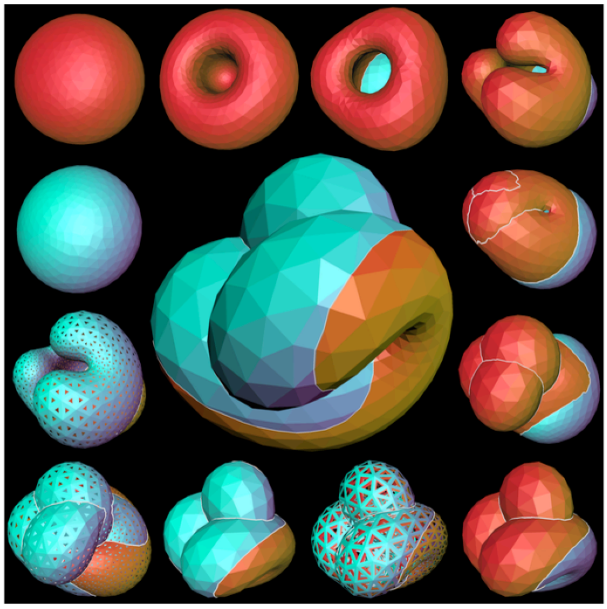
J. Sullivan, G. Francis, and S. Levy, "The Optiverse", Math & NCSA, U. Illinois, © 1998

Boy
Tableau
n=3



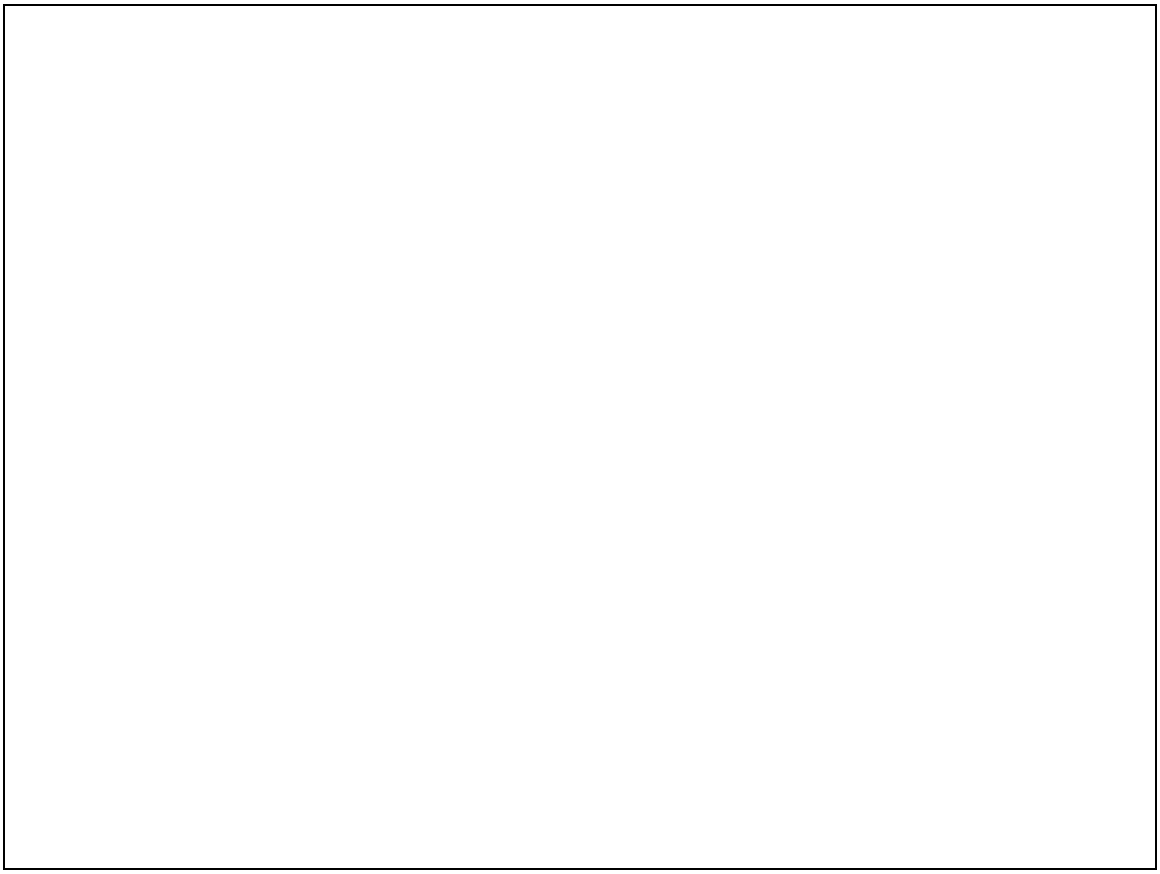
from George K. Francis, “A Topological Picturebook”, Springer-Verlag, 1987

Boy
Montage
n=3



J. M. Sullivan, G. Francis, and S. Levy, "The Optiverse", Math & NCSA, U. Illinois, © 1998

***Bernard Morin "sees" his Sphere Eversion***



# Morin Eversion

1967

2000

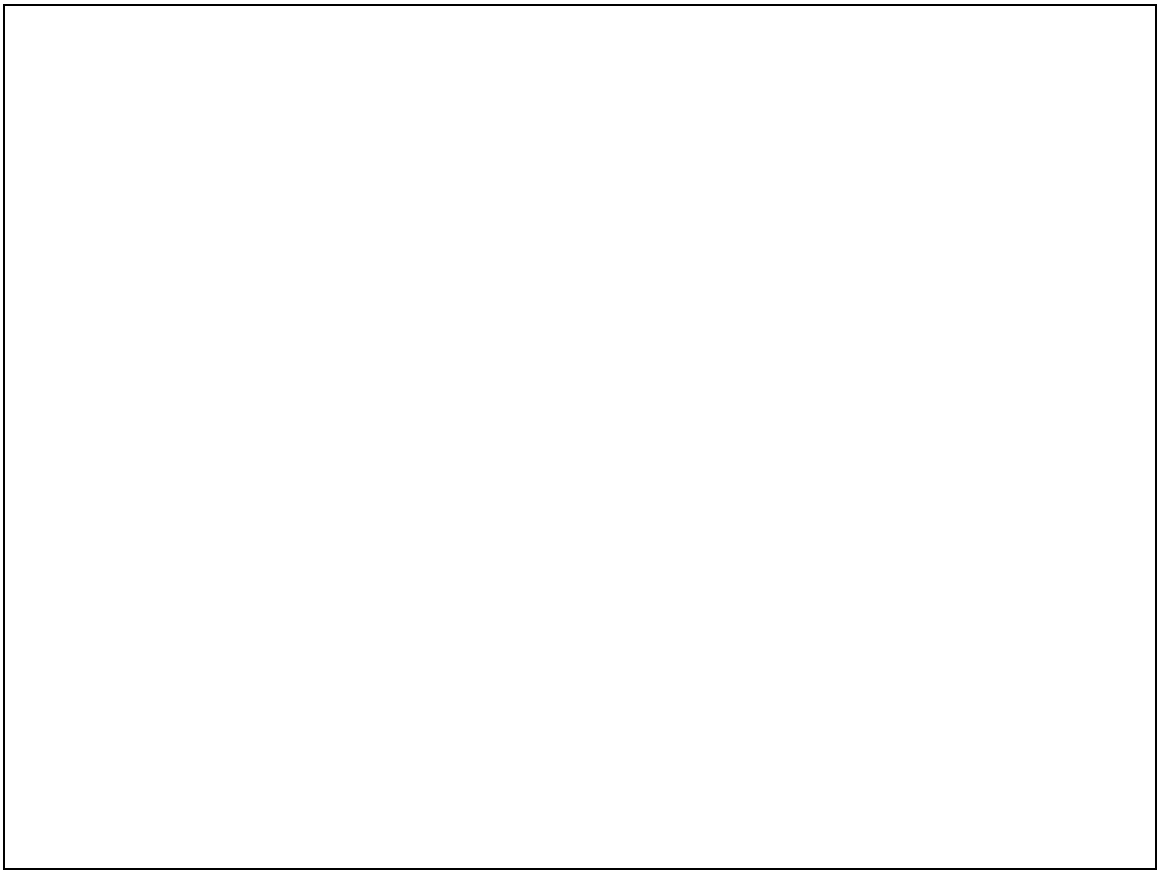
Bernard Morin at Maubeuge, France, 20 September 2000

Bernard Morin

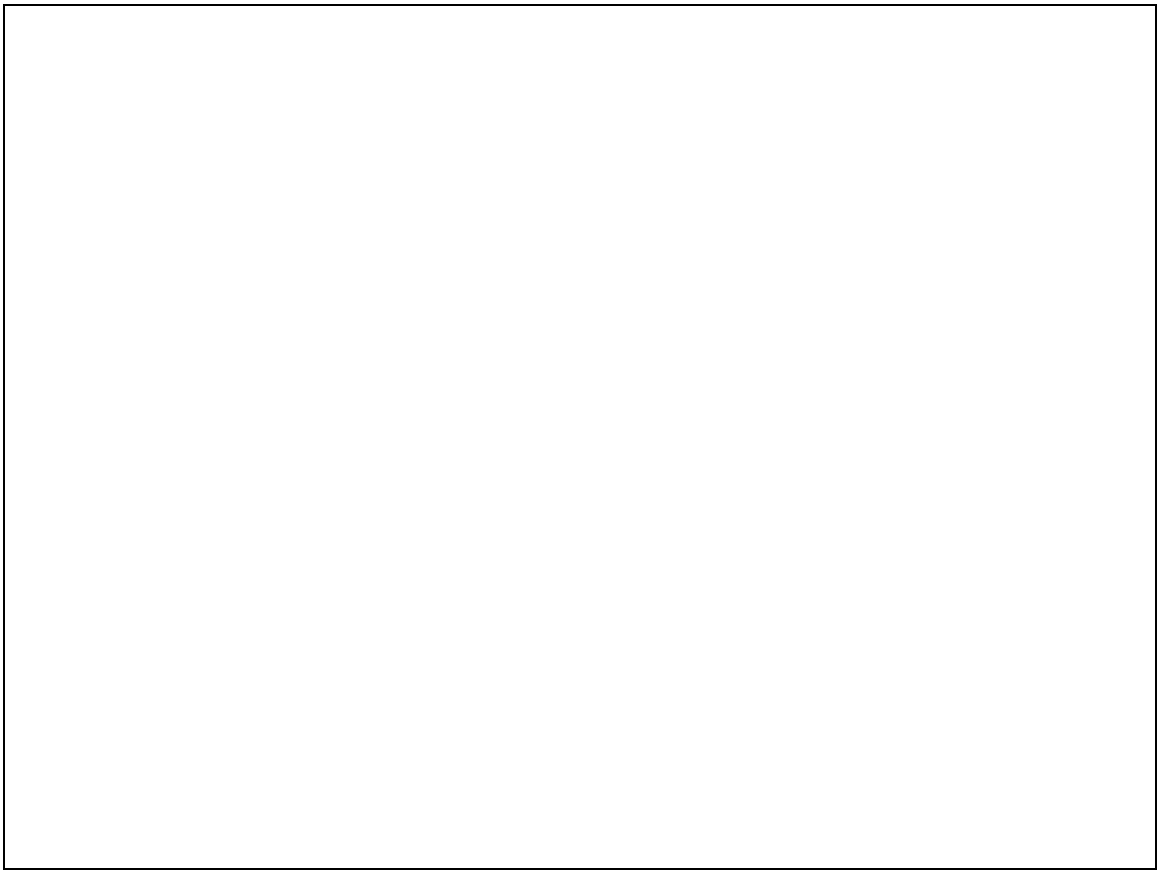
looking at
Stuart Dickson's stereolith models of
John Sullivan's *Minimax Eversion,*
Maubeuge, France, September, 2000

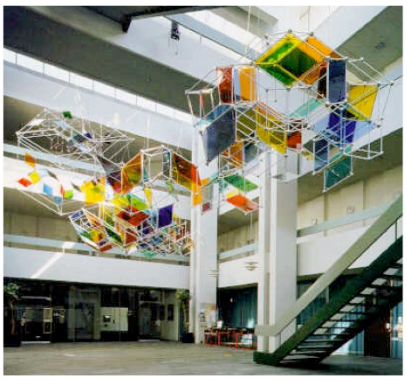
# Dickson's Stereoliths

***Five Morin-Apéry RTICAs***

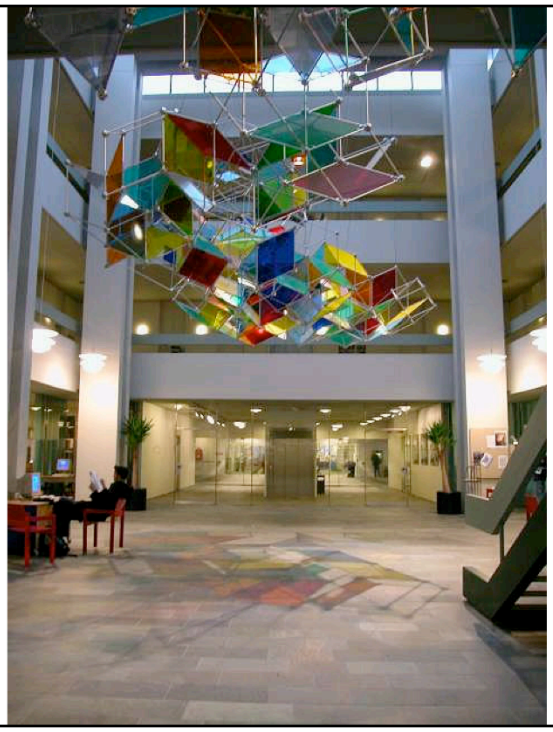


## *Quasicrystal Frameworks*

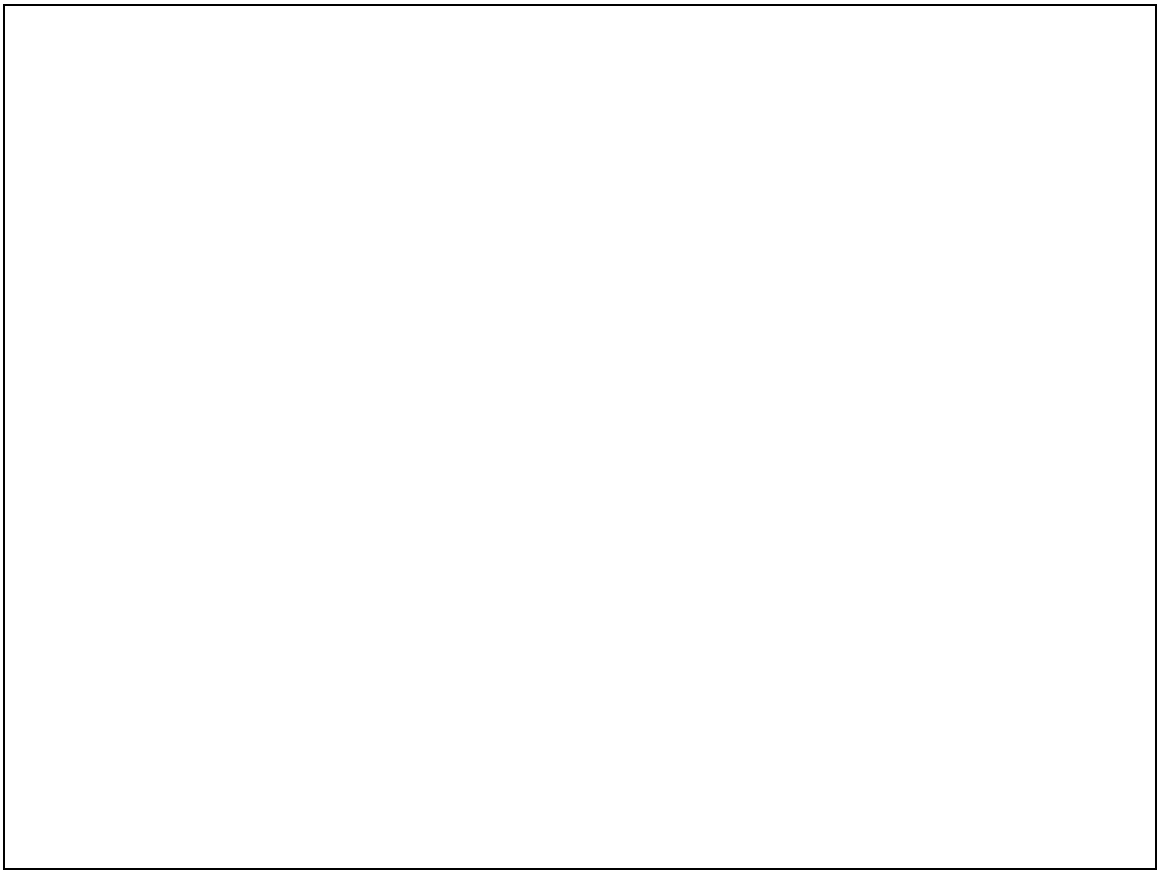


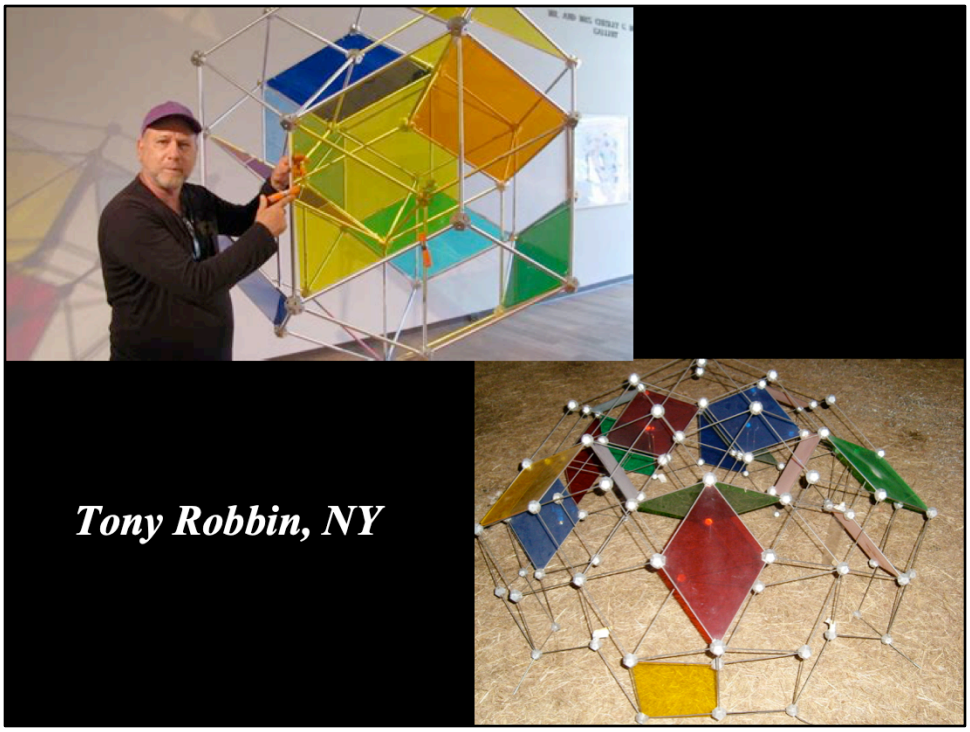


**COAST**
**Tony Robbin 1994**
**Center for Arts Sciences and Technology at the Danish Technical University**
**Erik Reitzel - engineer**
**RCM Precision - fabrication**
**Poul Ib Hendriksen - photos**



COAST was destroyed by
a misguided administrator






The original was destroyed, the model survives

*With thanks*
*to all in the*

*illiMath Collective*
*1977-2022*

## Collaborators, co-authors, friends

Teachers:
PLATO 1977-1980:
**Judy and Bruce Sherwood**
Dept of Industrial Design:
**Norm MacFarland, Ed Zagorski, Vivian Faulkner-King**
Student Assistants:
UIMATH.Applelab:1983-1994
**Jim Bailey Ferrell Wheeler**
**Ted Emerson Cary Sandvig**
REL/CAVE/grafiXlab:1988-2000
**Ray Idaszak Glenn Chappell**
**Chris Hartman Alex Bourd**
**Ulises Cervantes-Pimentel**
**John Estabrook Matt Hall**

Colleagues:
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**Bob Illyes Graham Evans**
NCSA, Urbana 1986-2000:
**Donna Cox Carl Hoyer,**
**Bob Patterson Jeff Carpenter.**
EVL, Chicago 1987-1998:
**Dan Sandin Tom DeFanti**
**Maxine Brown Ellen Sandor**
**Dana Plepys Dave Pape**
**Carolina Cruz-Neira**
Geometry Center 1989-1997:
**Pat Hanrahan, Charlie Gunn, Stuart Levy, John Sullivan**

Virtual Environments:
CAVE, CUBE, CANVAS (ISL) **Ben Schaeffer, Jim Crowell, Camille Goudeseune, Hank Kaczmarski**
DiVE (Duke) **Rachael Brady, David Zielinsky**
Portal (TUB) **Samy Khedem, John Sullivan, Steffen Weissman**

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## Research Experiences for Undergraduates

Audible Sketchpad (NCSA 1998-2000): **Ande Croll , Jessica Jackson, Doug Nachand, Bob Pinta, Ben Shanbaum, Paul Whitaker, Matt Woodruff.**
illiMath2001 (VIGRE): **Ben Bernard, Ben Farmer, Mark Flider, Doug Nachand, Alison Ortony, Lorna Salaman, Ben Shanbaum, Robert Shuttleworth, Matt Woodruff.**
illiMath2002(VIGRE): **Amit Chatwani, Ben Farmer, Abdul Hamide, Brad Henry, Wendy Hubbard, Yana Malysheva.**
PyCube2004 (Math Dept): **William Baker, Blair Flicker, Emily Gunawan, Greg Stanton, Brett Witt.**
illiMath2006 (REUsite): **Dave Bergman, Nicholas Duchnowski, Emily Echevarria, Matt Gregory, Paul Prue, Chris Rainey, Mimi Tsuruga, Abby Watt.**
illiMath2008 (REUsite): **Chase Boren, Will Davis, Abdul Dakkak, Geoff Ehrman, Lisa Hickock, Sam Ostling, John Pacey, Katie Poon, Liz Rogers.**
illiMath2010 (REUsite): **Chris Bisom, Ian Markwood, Dan Rajchwald, Justin Schirle**
Associated Mentors: **Robert Acar (Puerto Rico), Peter Brinkmann (CCNY), Ulises Cervantes-Pimentel (WRI), Elizabeth Denne (Harvard), Abdul Dakkak (WRI), Paul McCreary (Evergreen), Mike Pelsmajer (IIT), Karen Shuman (Grinnell), Rose Marshack (ISU), Tony Robbin (NY), Jeff Weeks (NY).**

## Projects in the Illinois Geometry Lab

***Stability of Quasicrystal Frameworks***
Spring 2013: **Alex Burnley, Chong Han**
Fall 2023: **Keran Huang, Natchiket Joshi, Jonathan McGreal**
Spring 2014: **Zachary Miksis, Daniel Pugliese, Joseph Zeller**
Spring 2017:
The 3D project: **Yijing Chen, Arturo Guerrero, Sasha Lamtyugina, Yi "Lisa" Li**
The 2D project: **Pranav Bhardwaj, Manting Huang, Tejo Nutalapati, Sung Jib Kim**
Graduate Mentor**:** **Eliana Duarte**

***Computer Visualization in Experimental Mathematics***
Fall 2019: **Joshua Holder, Xiaomin Li, Zhuoyun "Doris" Wang, Jinlin Xu**
Graduate Mentors: **Daniel Carmody, Karthik Vasu**

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http://new.math.uiuc.edu/

soon

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