

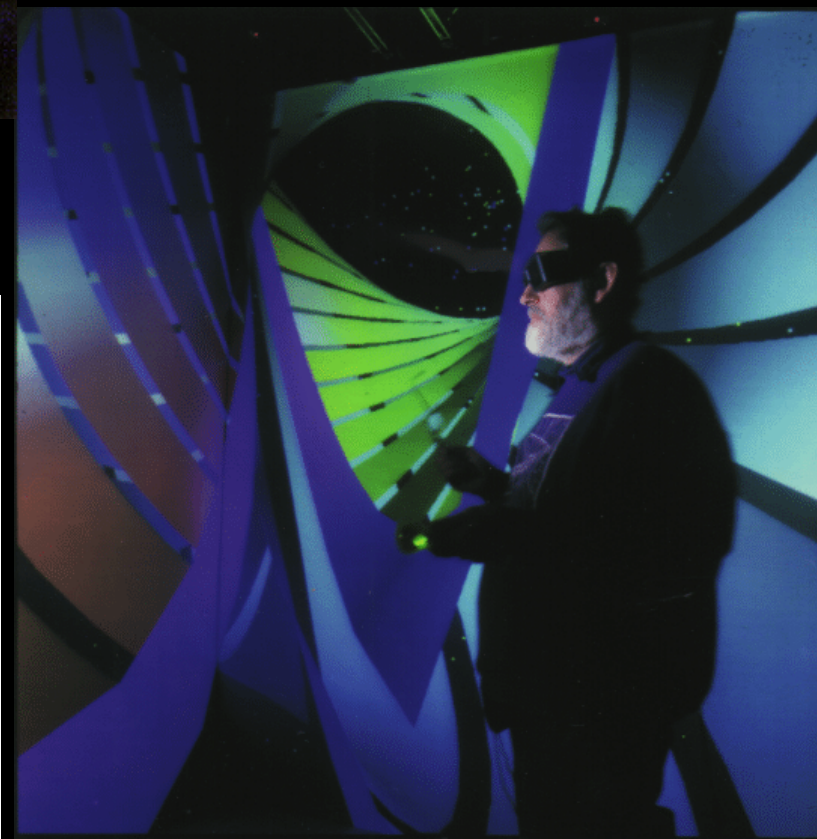
mathARTmath:

a geometrical puppetshow
at Kansas State University

on 10 March 2022

by George K. Francis
University of Illinois

<http://new.math.uiuc.edu/puppetshow>



Program

The Prelude: From Pythagoras to Conway

Cover Girls from Villendorf and the NCSA star in...

RTICA “venus”: François Apéry’s Romboy Homotopy

... and in PLATO’s CAVE at the NCSA

Chapter -1:

Topological Picturebook (1987) and the Geometrical Puppetshow (20??)

Linear Perspective, Klein Bottles, Figure-8 Knots, etc

Tutorial: Chalk on Blackboard

Zeeman’s Contractible but not Collapsible Surface

RTICA “dunce”: John Dalbec’s Contracting Duncelhat

CAVE Operas:

Sphere Eversions and Other CAVE Operas

RTICA “five”: Bernard Morin’s Five Eversion Precursors

Preview: Tony Robbin: Quasicrystalline Art and Math.

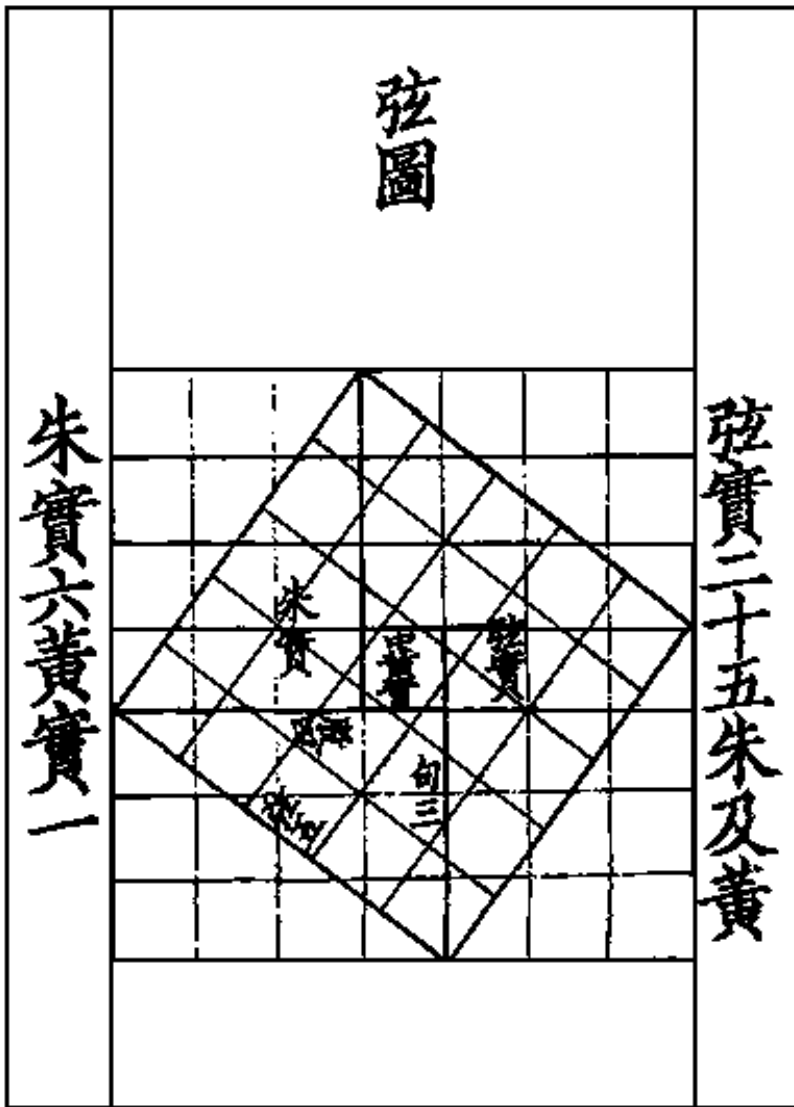
The Credits: The illiMath Collective

The Encore: Are there any Questions?

RTICA= Real-Time Interactive Computer Animation

NCSA = National Center for Supercomputing Applications

CAVE = CAVE Automatic Virtual Environment



On the throne of Isis nursing Horus we found depicted the classical Geometrical Demonstration of Pythagoras' Theorem

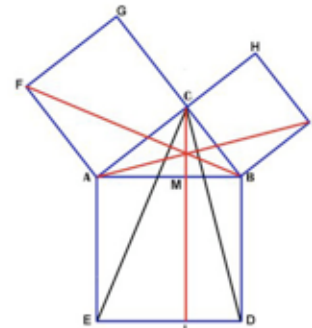
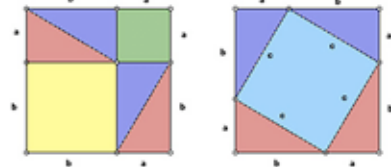
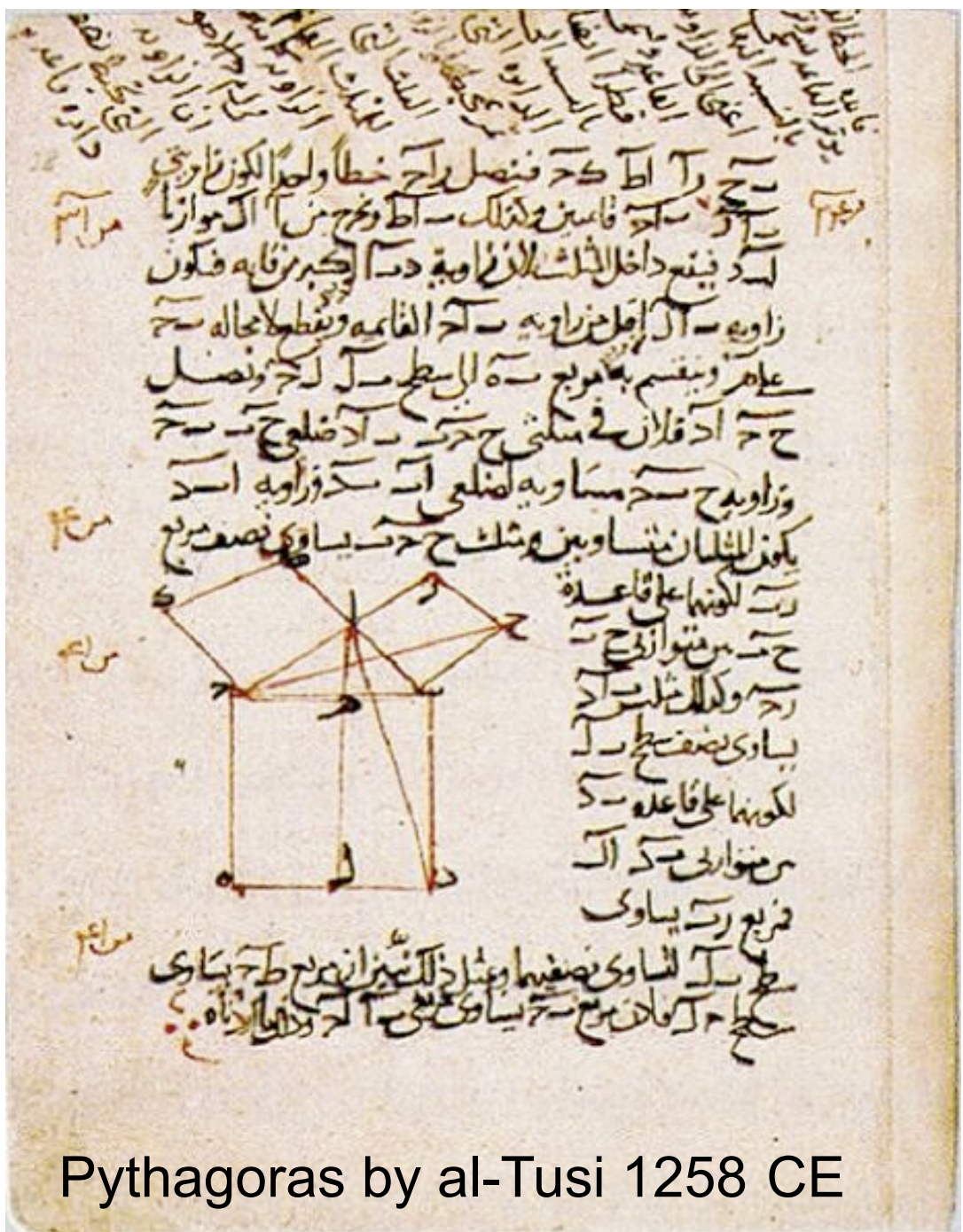


Figure 1 - The figure of proof upon which the original 47th Problem of Euclid is based. This figure is also called "The Bride's Chair". The outline of this figure is used Symbolically in Freemasonry.



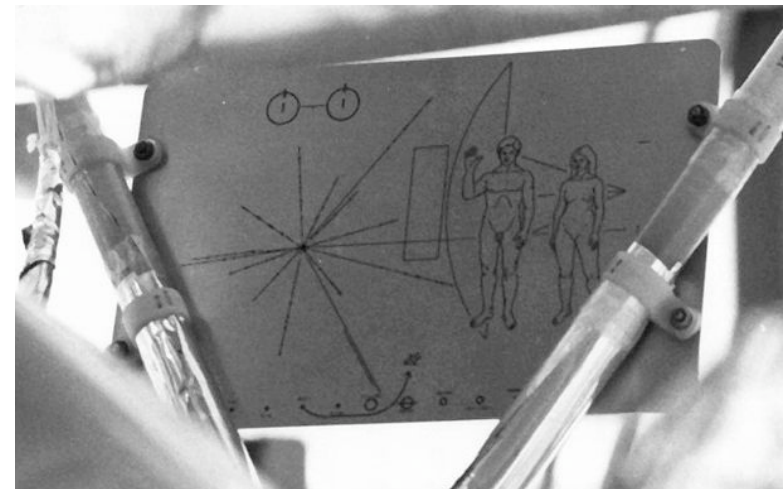
Pythagoras' Theorem according to Google



Pythagoras by al-Tusi 1258 CE

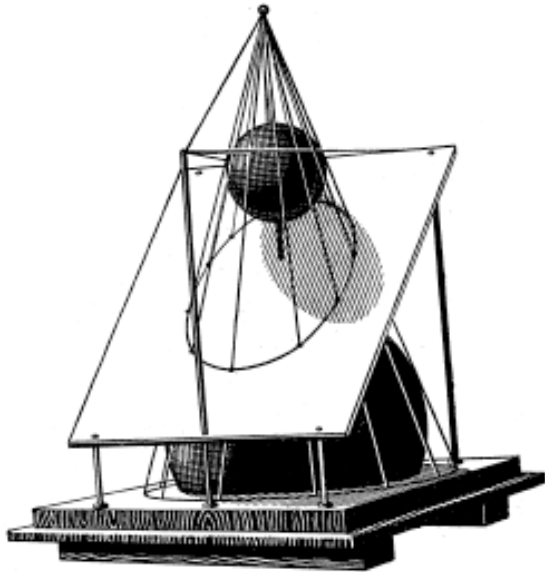
The picture for Euclid's proof was chosen for the plaque on Pioneer 10 not so much to prove that we knew the theorem, but that we knew how to prove it


Yuri Rainich,
 1955



VERZEICHNIS
MATHEMATISCHER
MODELLE

SAMMLUNGEN H. WIENER UND P. TREUTLEIN



AUS DEM VERLAG  VON B.G. TEUBNER
IN LEIPZIG UND BERLIN 1912

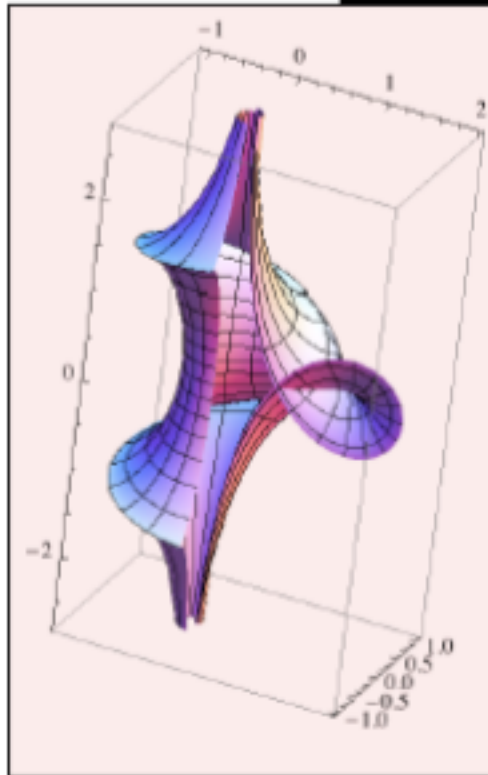
Dandelin's Theorem x 3



Tori Corkery, Math Calendar 2009

Kuehn's Surface

$$x[u_-, v_-] := \frac{2 (\cos[u] + u \sin[u]) \sin[v]}{1 + u^2 \sin^2[v]}$$
$$y[u_-, v_-] := \frac{2 (\sin[u] - u \cos[u]) \sin[v]}{1 + u^2 \sin^2[v]}$$
$$z[u_-, v_-] := \log \left[\tan \left[\frac{v}{2} \right] \right] + \frac{2 \cos[v]}{1 + u^2 \sin^2[v]}$$



Scanned image

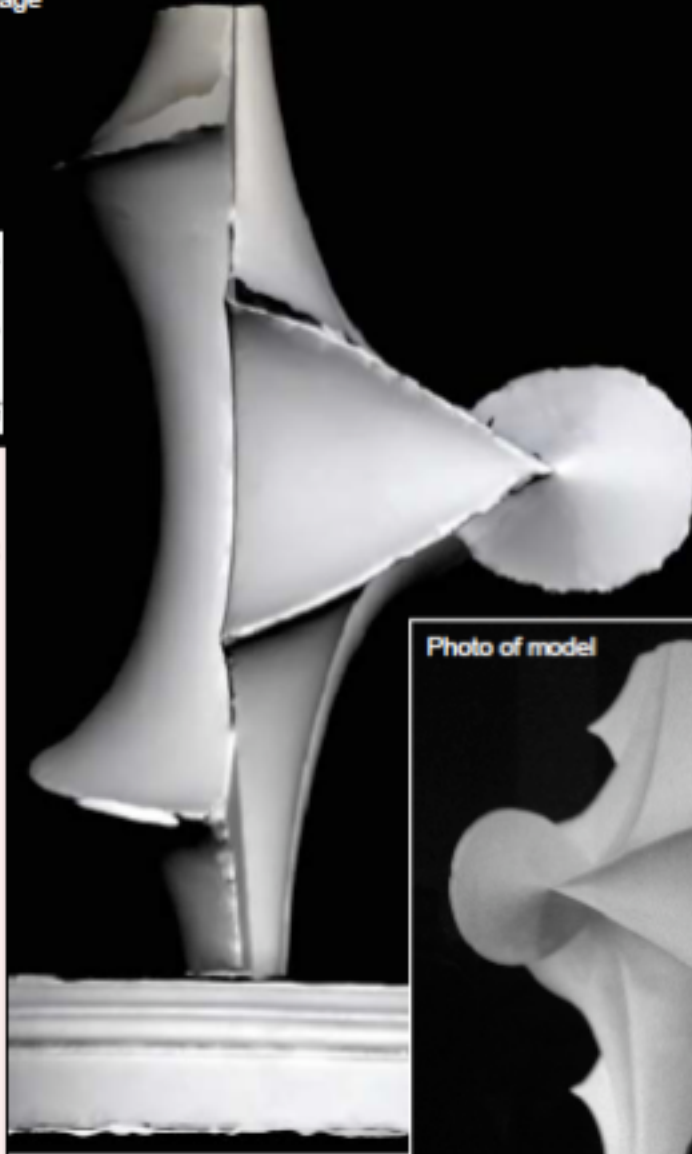
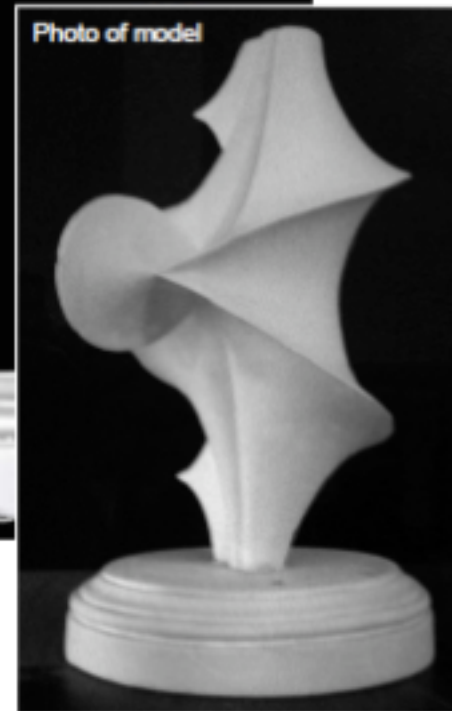
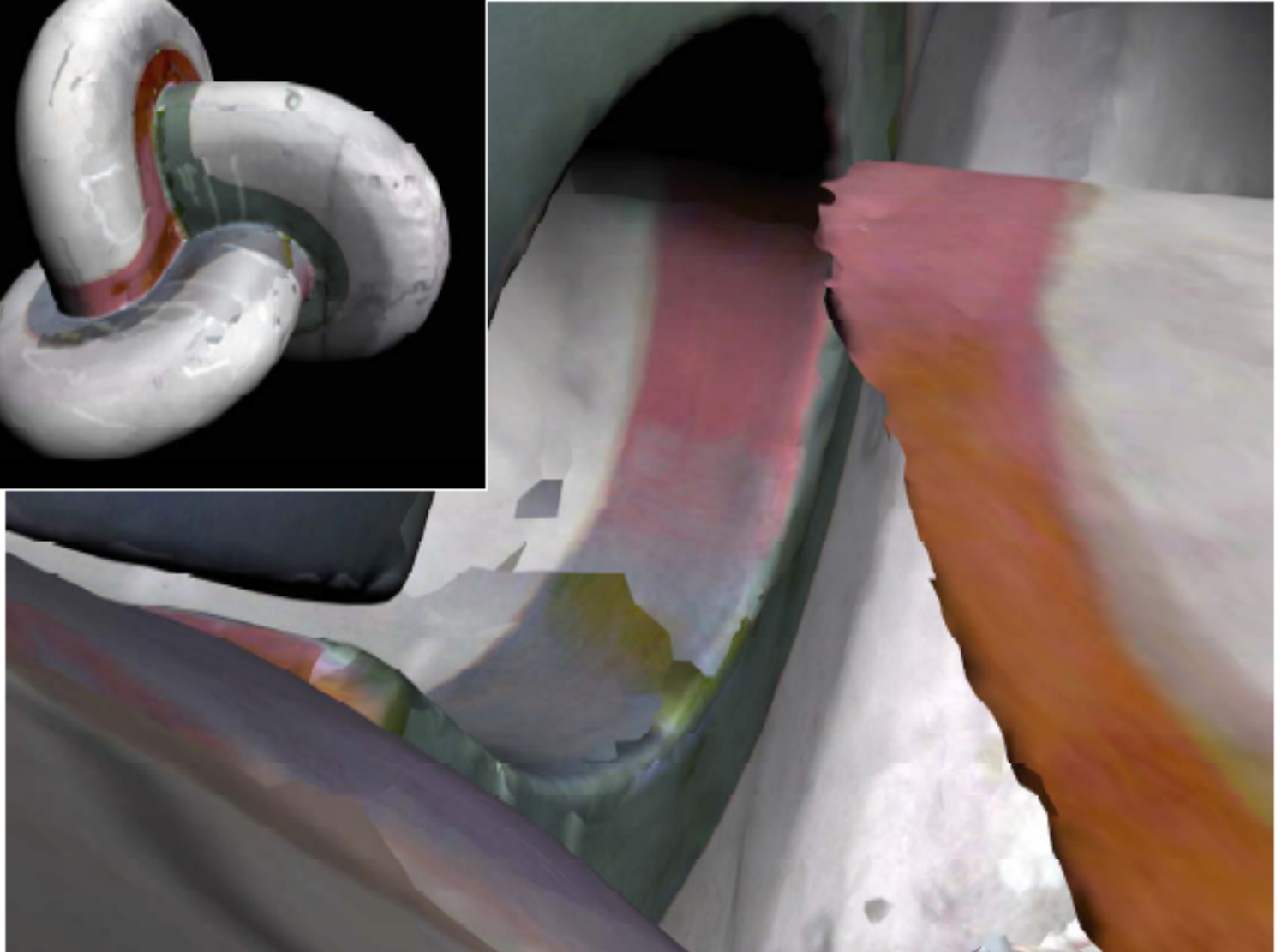
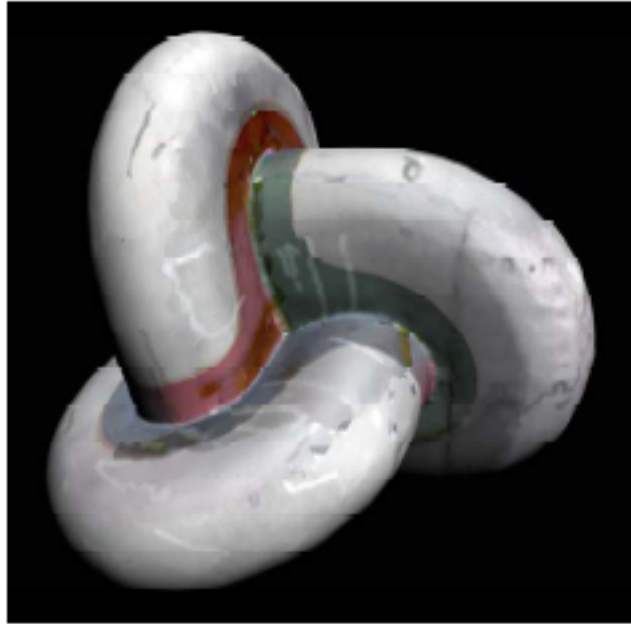


Photo of model



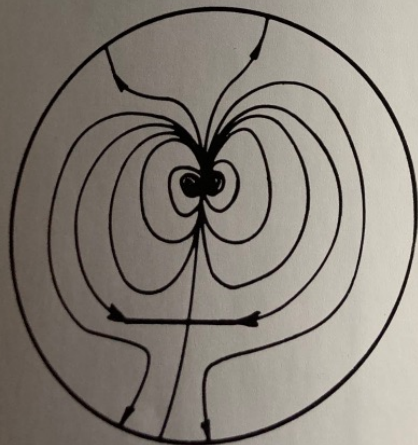


TOPOLOGY

FROM THE
DIFFERENTIABLE
VIEWPOINT

By John W. Milnor
Princeton University

Based on notes by
David W. Weaver



algebra

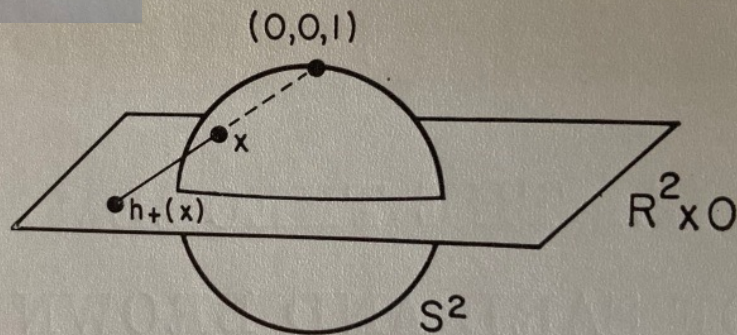


Figure 3. Stereographic projection

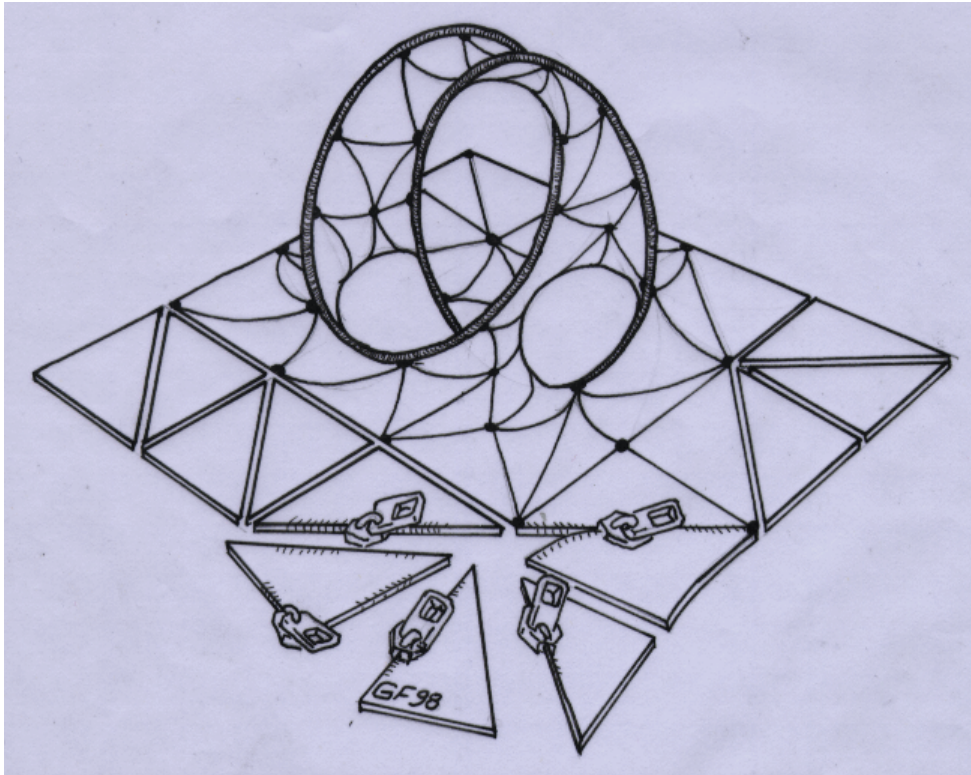
hood of the north pole. To see this we introduce the stereographic projection h_- from the south pole $(0, 0, -1)$ and set

Conway's ZIP Proof

Geore K. Francis and Jeffrey R. Weeks
Amer. Math. Monthly, 106 (May)1999

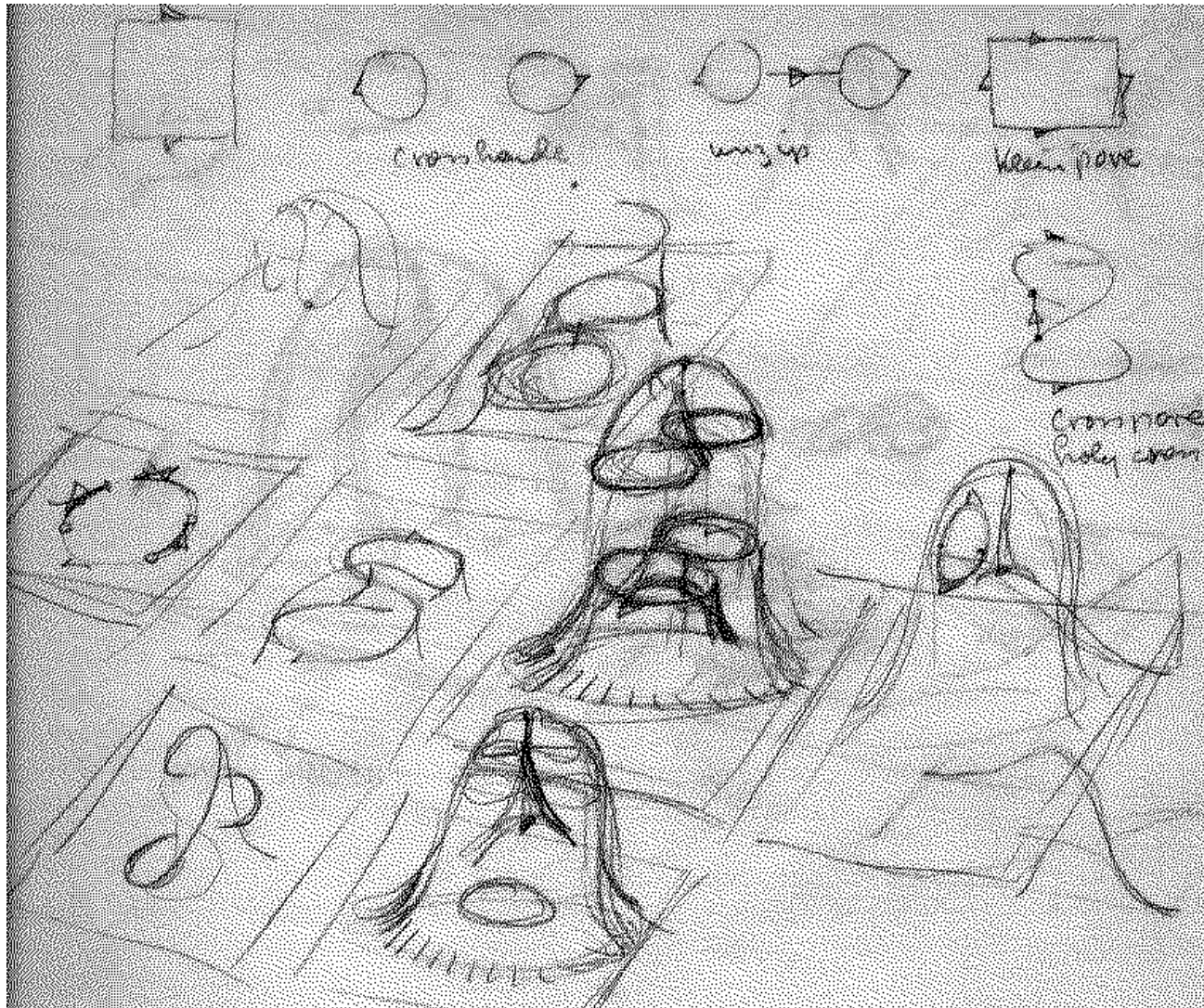
Surface = Sphere

with
Pores (holes)
Handles
Crosscaps
and
(unnecessary)
Crosshandles



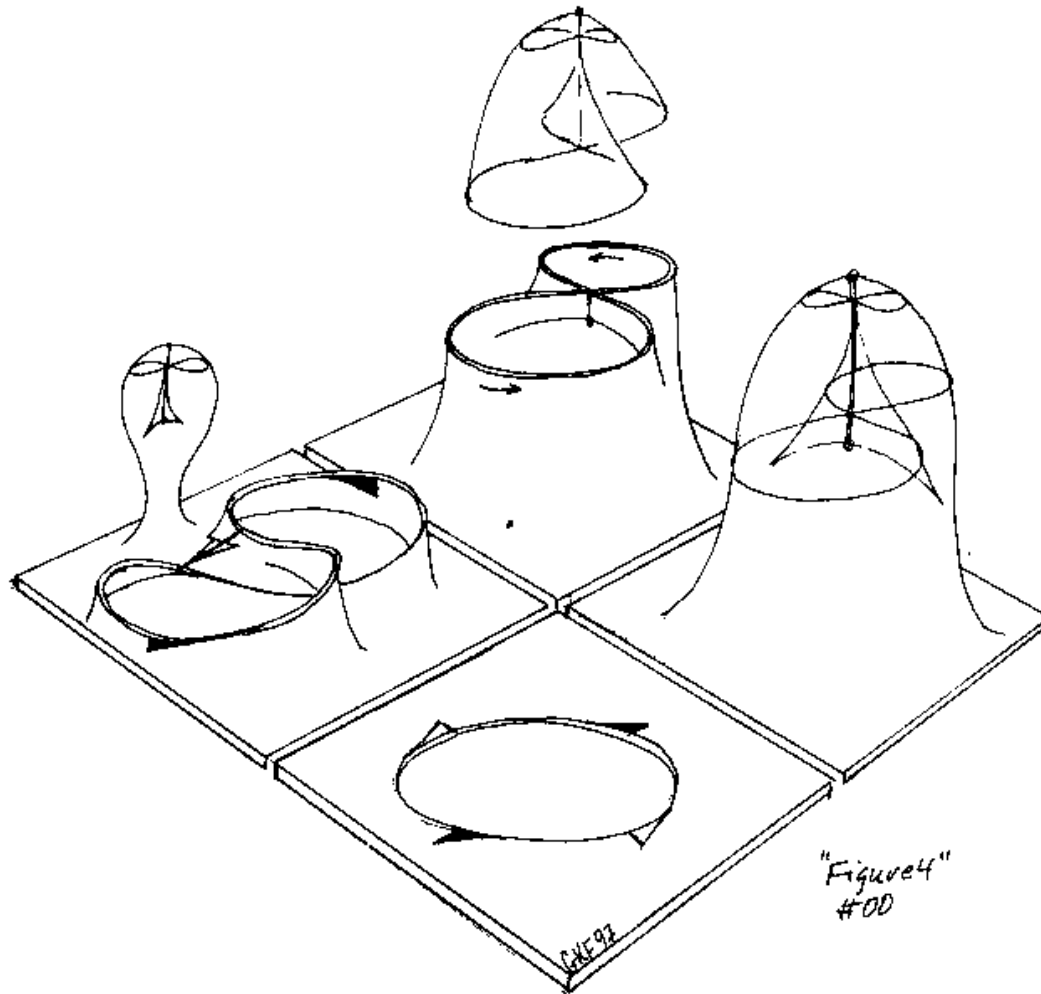
Triangulated Seifert surface spanning a Hopf link

Paper napkin sketches



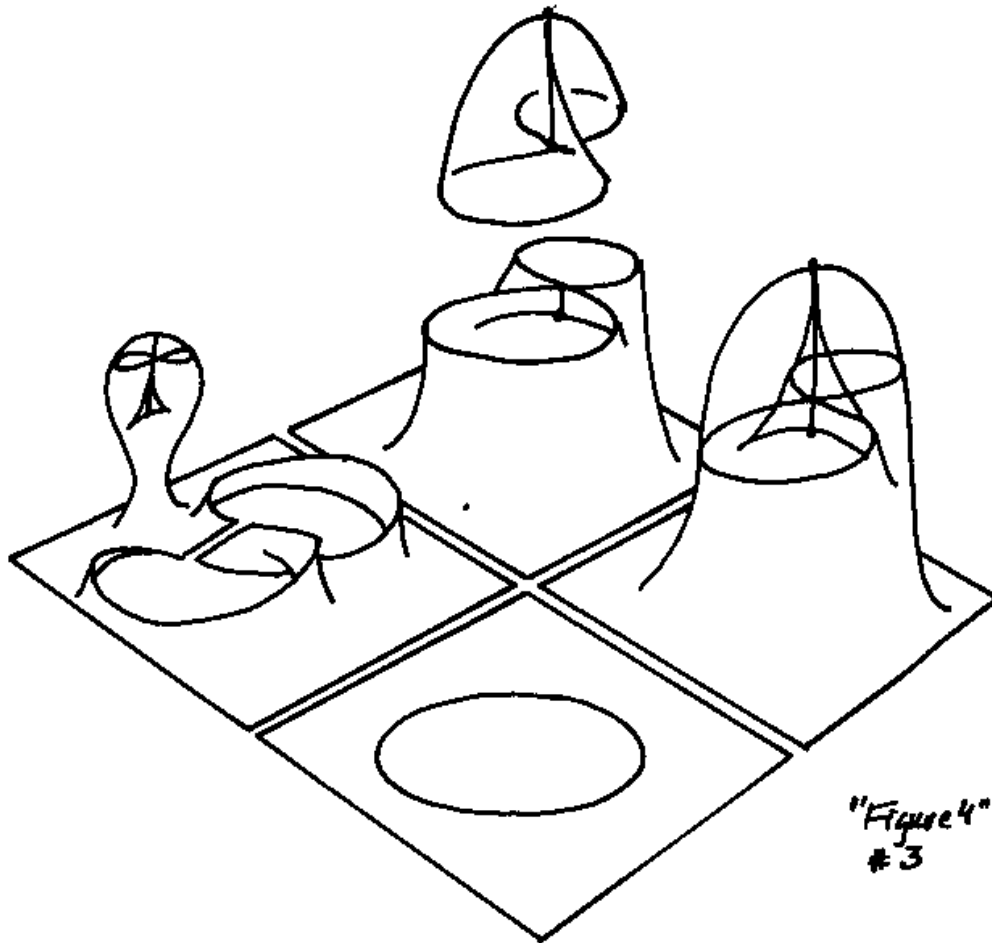
G.Francis and Jeff Weeks, "Conway's ZIP Proof, Amer Math Monthly, 1999

Crosscap one



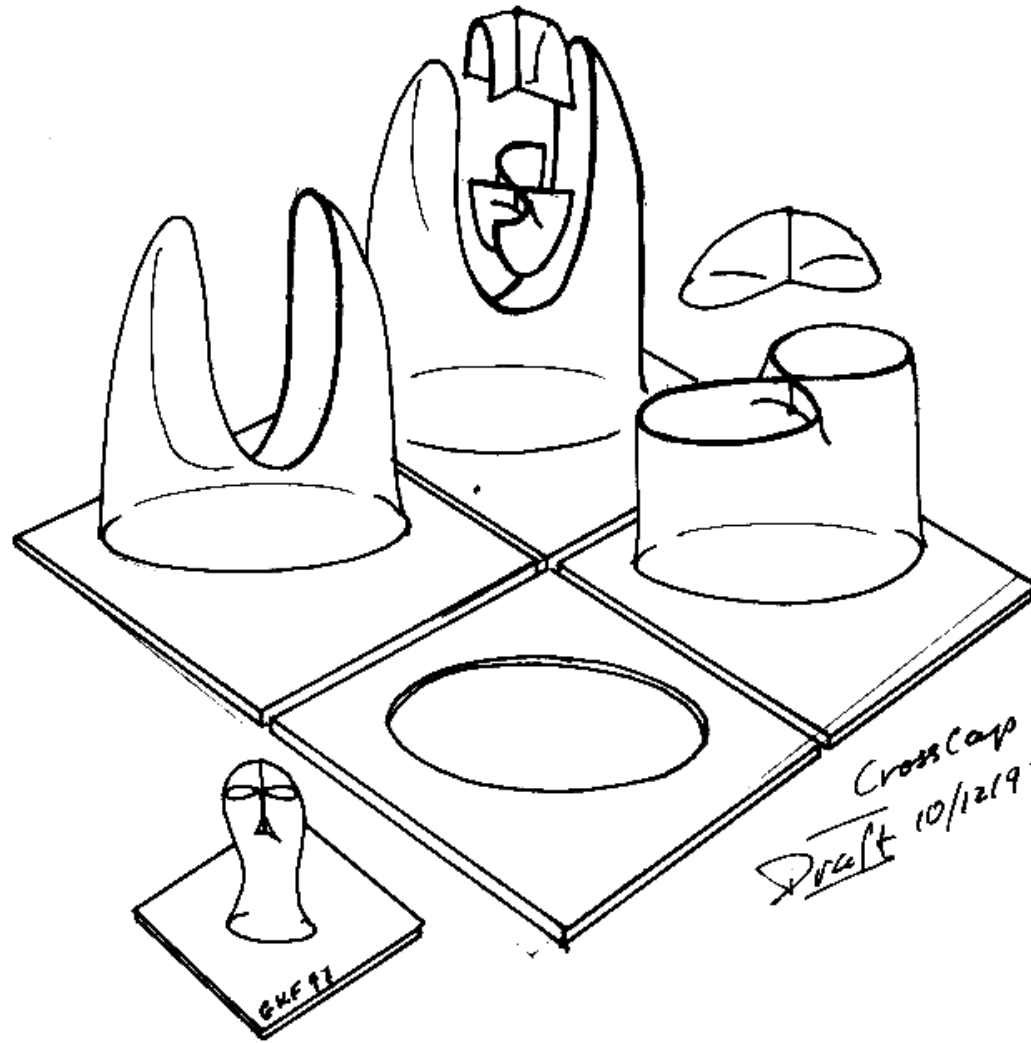
G.Francis and Jeff Weeks, "Conway's ZIP Proof, Amer Math Monthly, 1999

Crosscap two



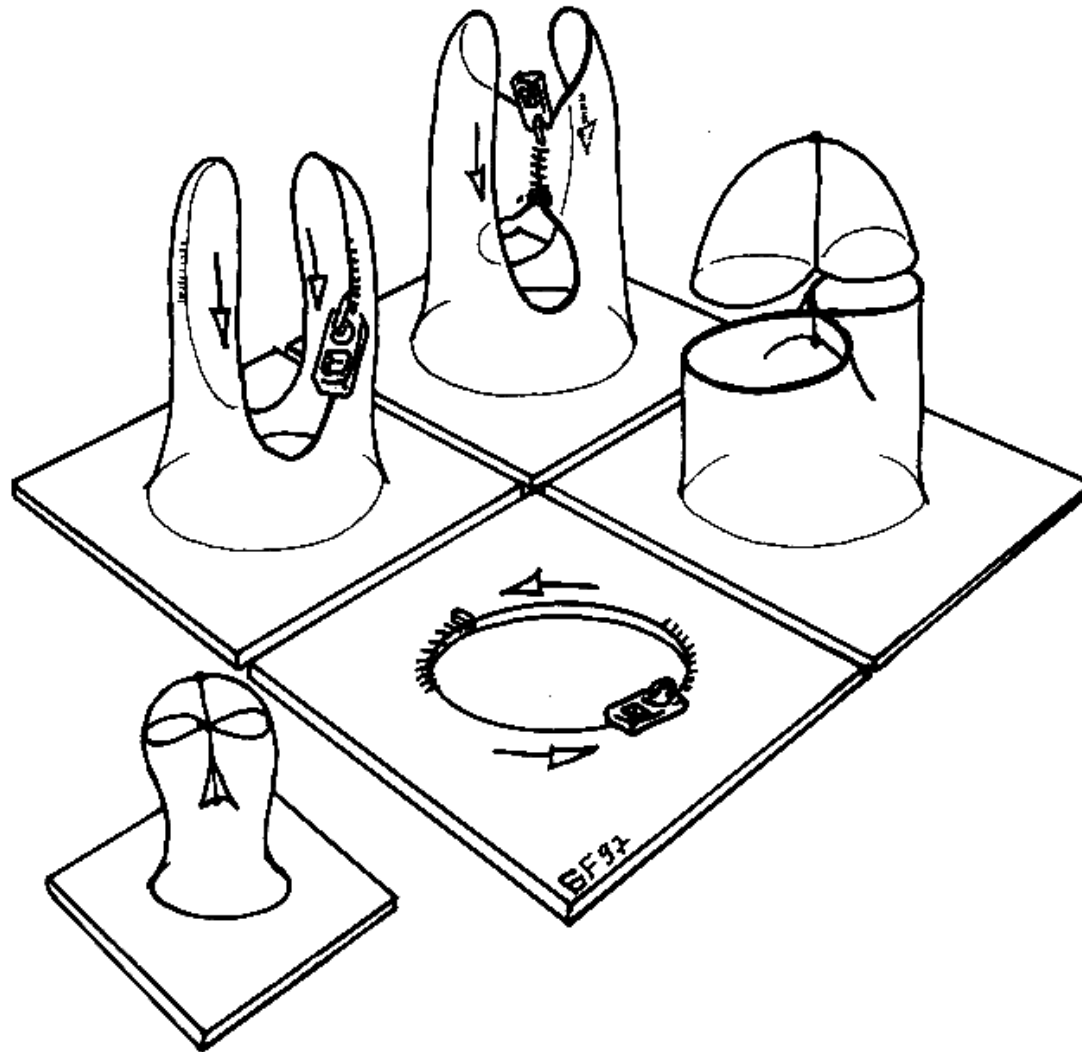
G.Francis and Jeff Weeks, "Conway's ZIP Proof, Amer Math Monthly, 1999

Crosscap three



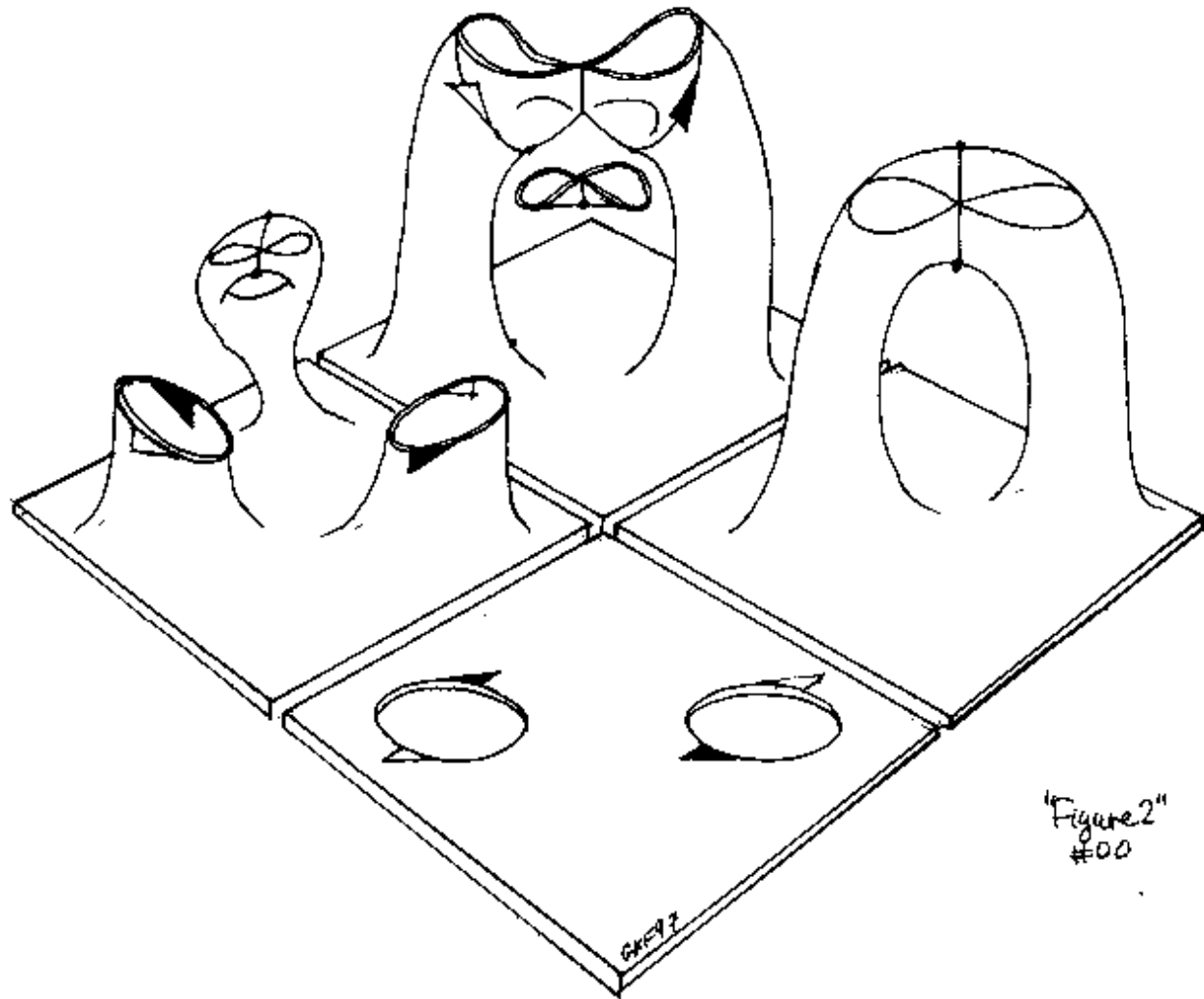
G.Francis and Jeff Weeks, "Conway's ZIP Proof, Amer Math Monthly, 1999

Crosscap final



G.Francis and Jeff Weeks, "Conway's ZIP Proof, Amer Math Monthly, 1999

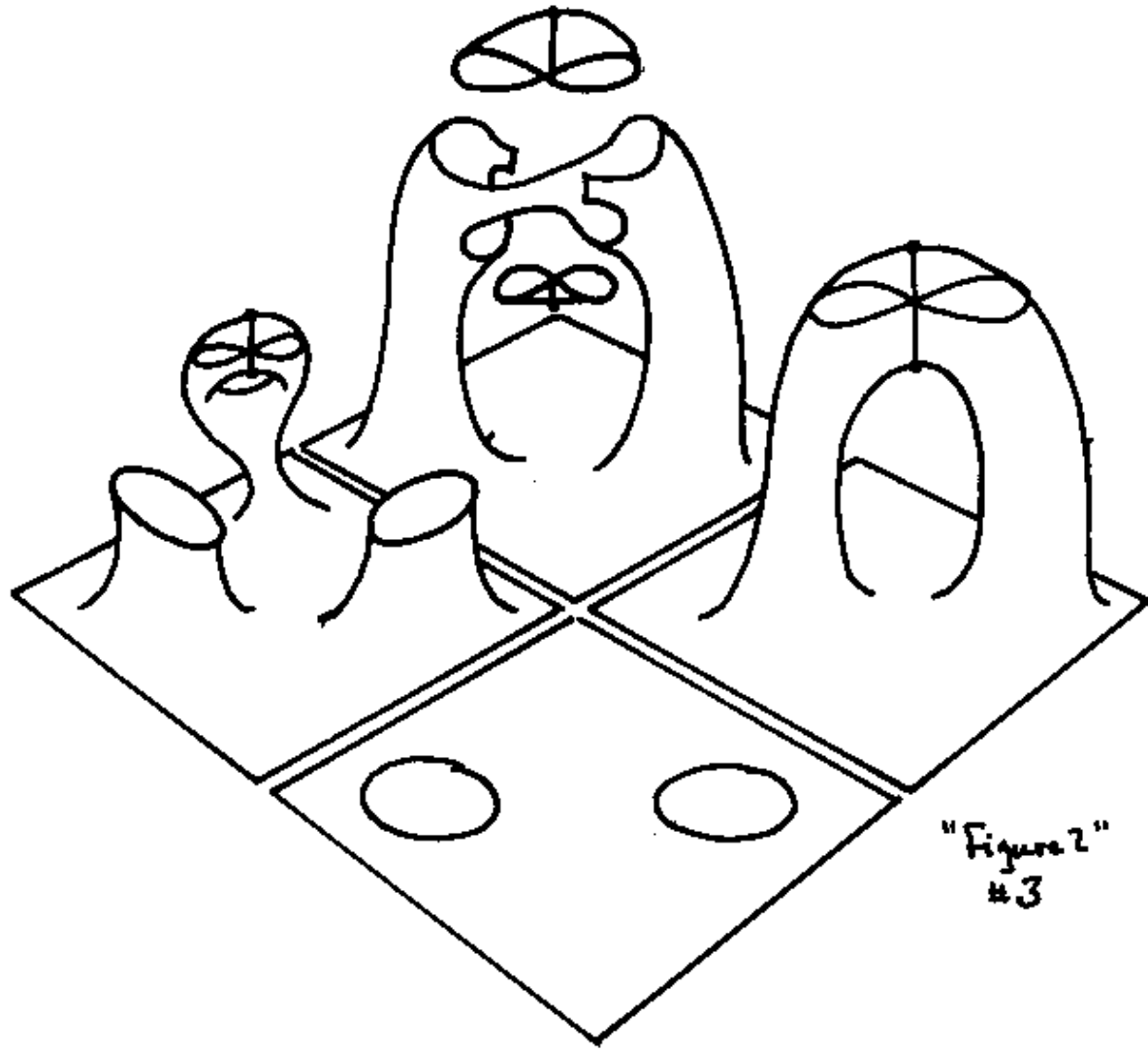
Crosshandle one



'Figure 2'
#00

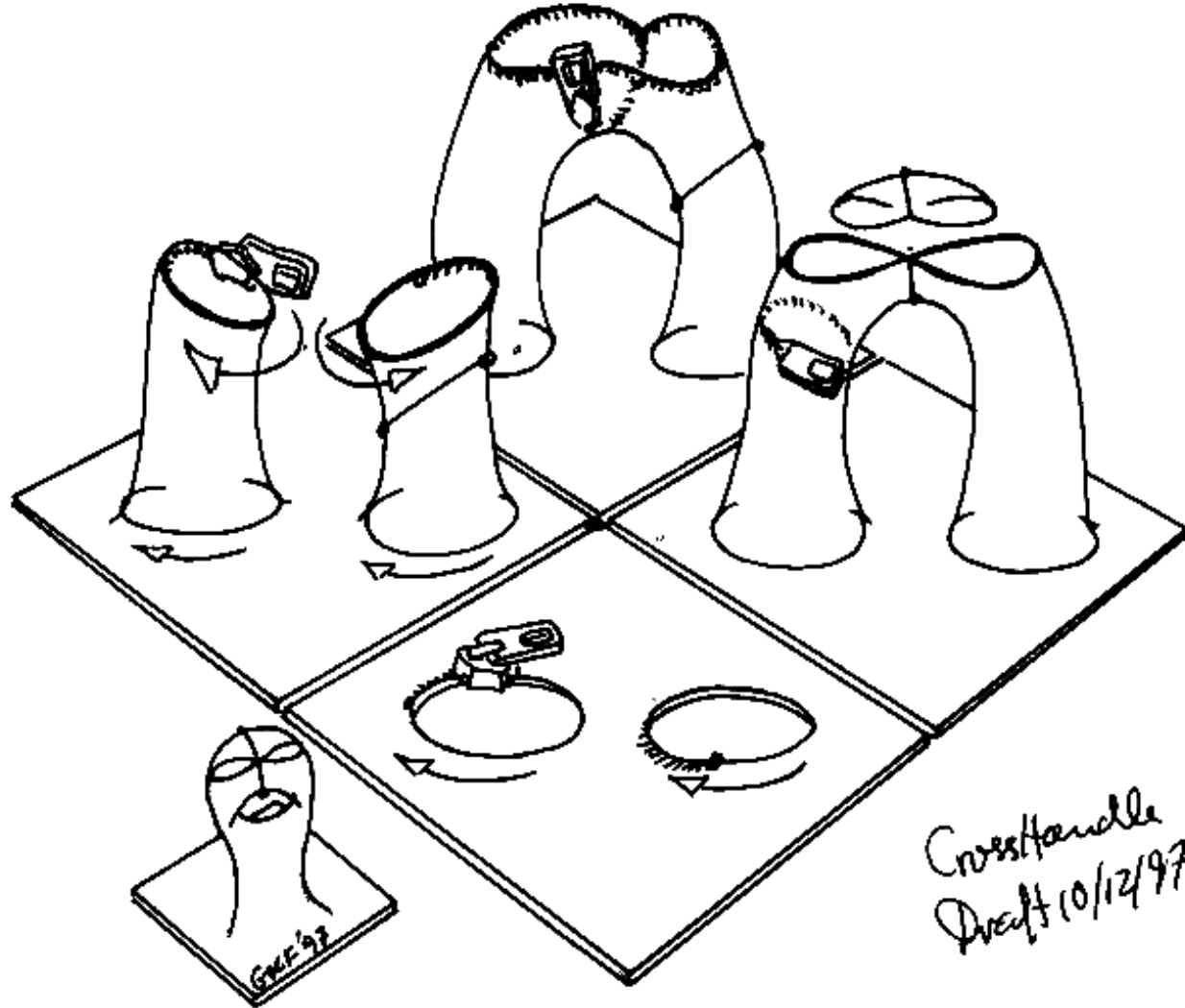
G.Francis and Jeff Weeks, "Conway's ZIP Proof, Amer Math Monthly, 1999

Crosshandle two



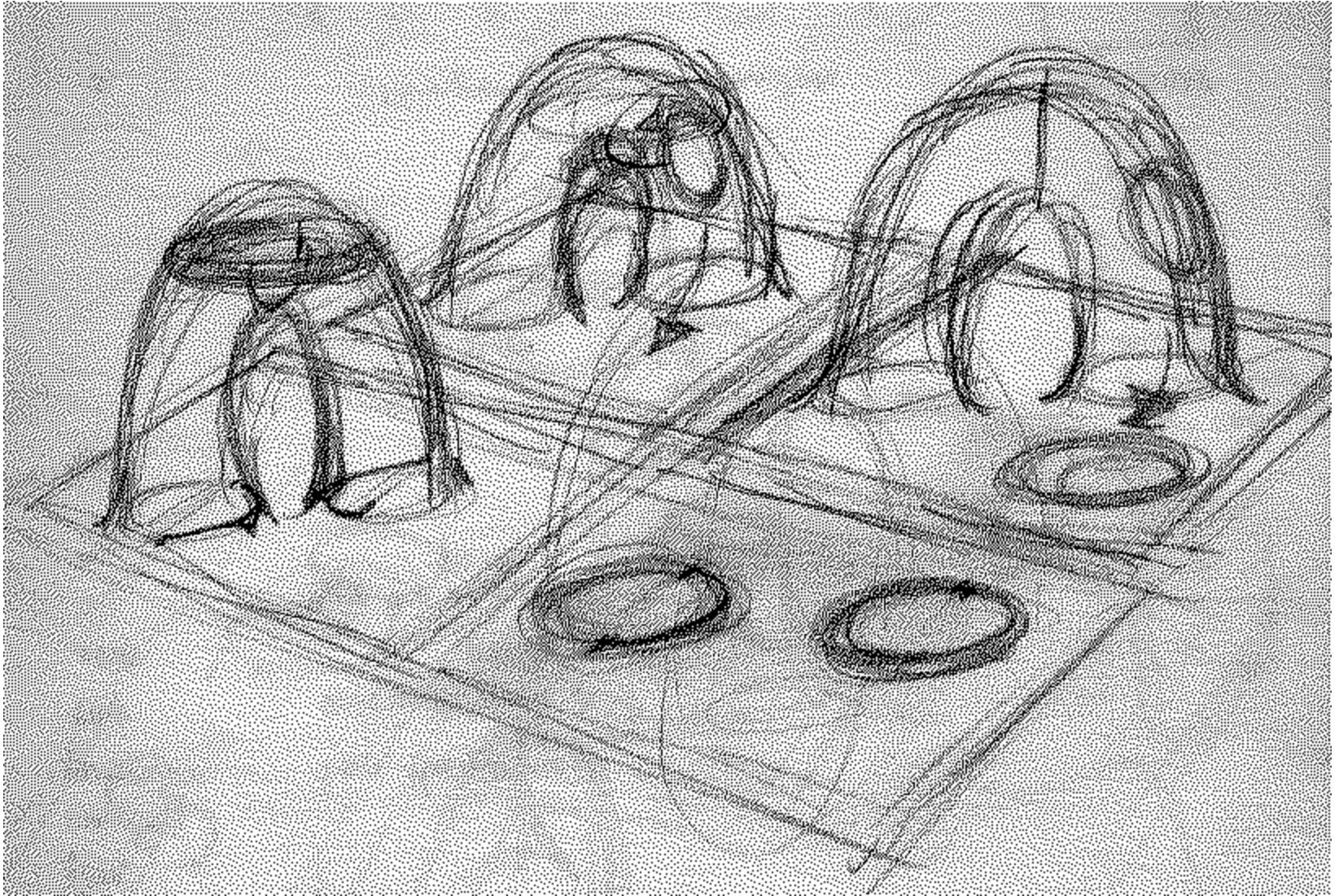
G.Francis and Jeff Weeks, "Conway's ZIP Proof, Amer Math Monthly, 1999

Crosshandle final



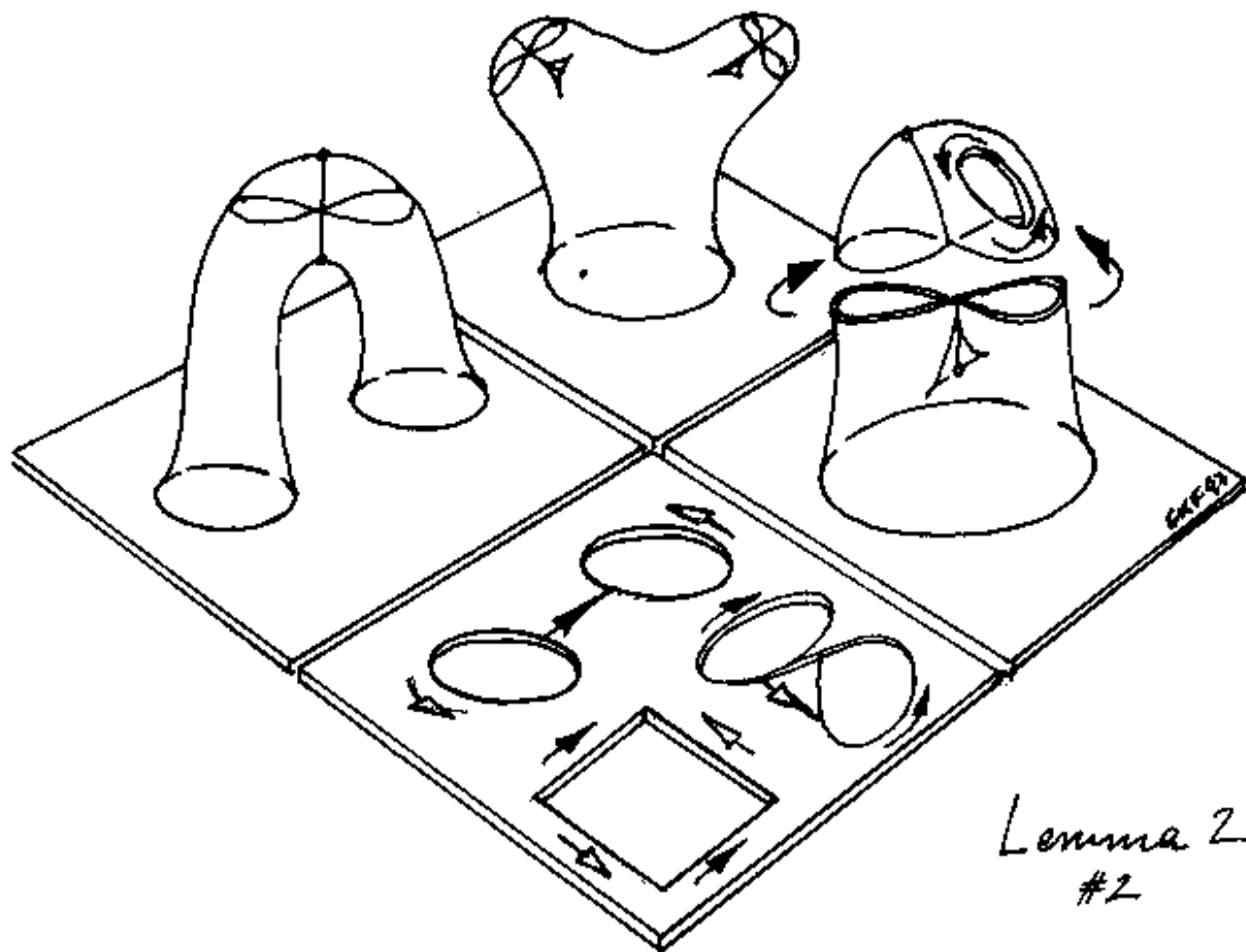
G.Francis and Jeff Weeks, "Conway's ZIP Proof, Amer Math Monthly, 1999

Kleinpore sketch



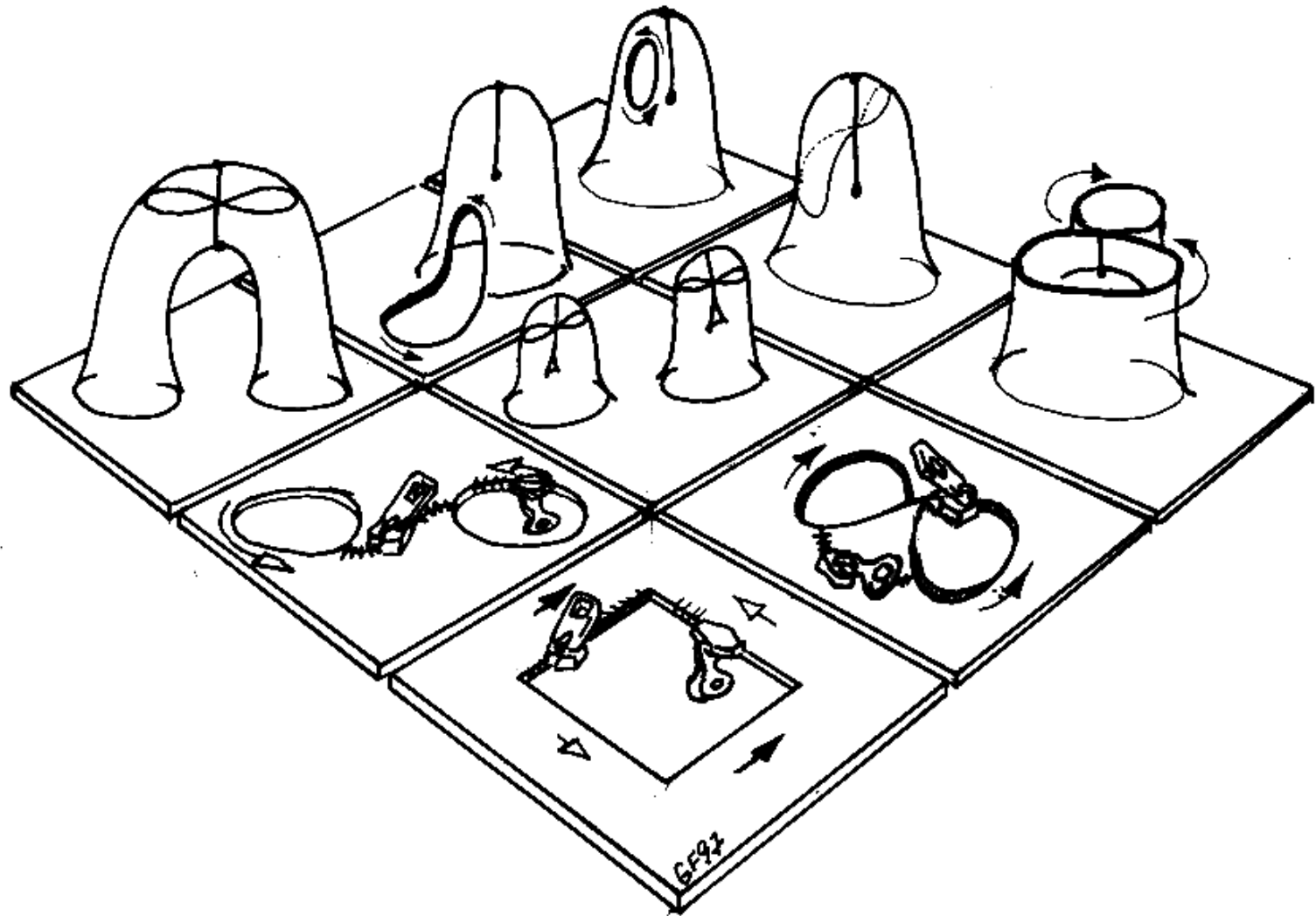
G.Francis and Jeff Weeks, "Conway's ZIP Proof, Amer Math Monthly, 1999

Kleinpore, second draft



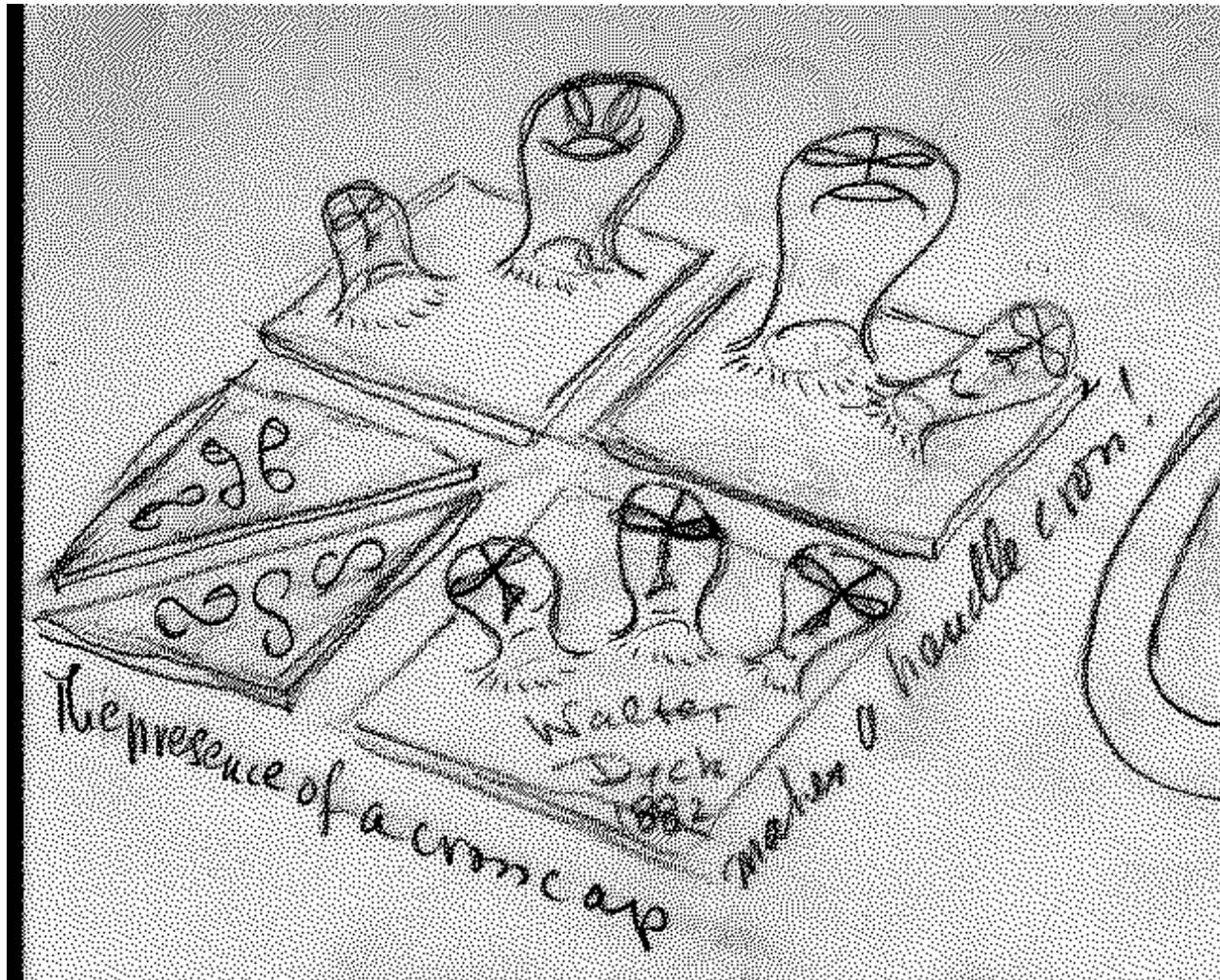
G.Francis and Jeff Weeks, "Conway's ZIP Proof, Amer Math Monthly, 1999

Kleinpore: 1 xhandle = 2 xcaps



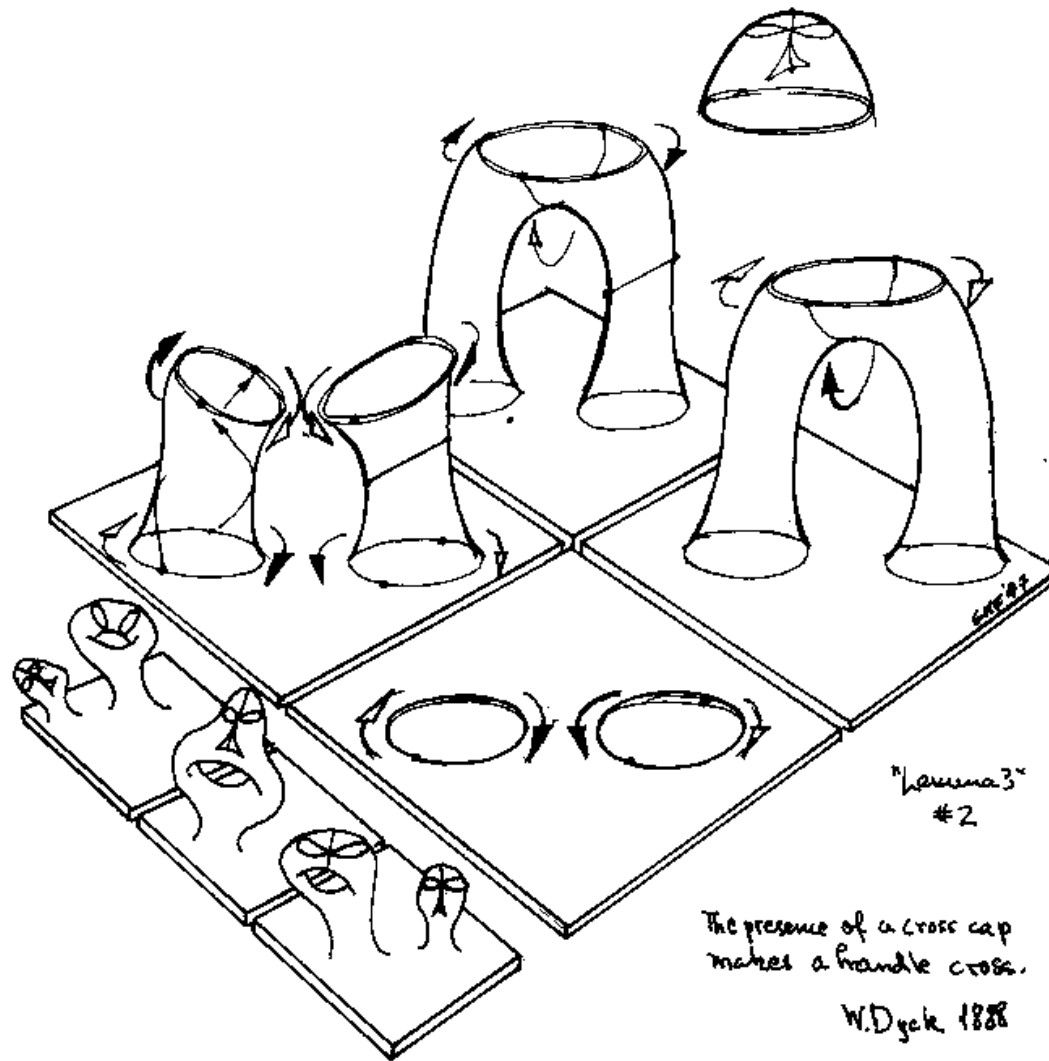
G.Francis and Jeff Weeks, "Conway's ZIP Proof, Amer Math Monthly, 1999

Dyck's Theorem, first sketch



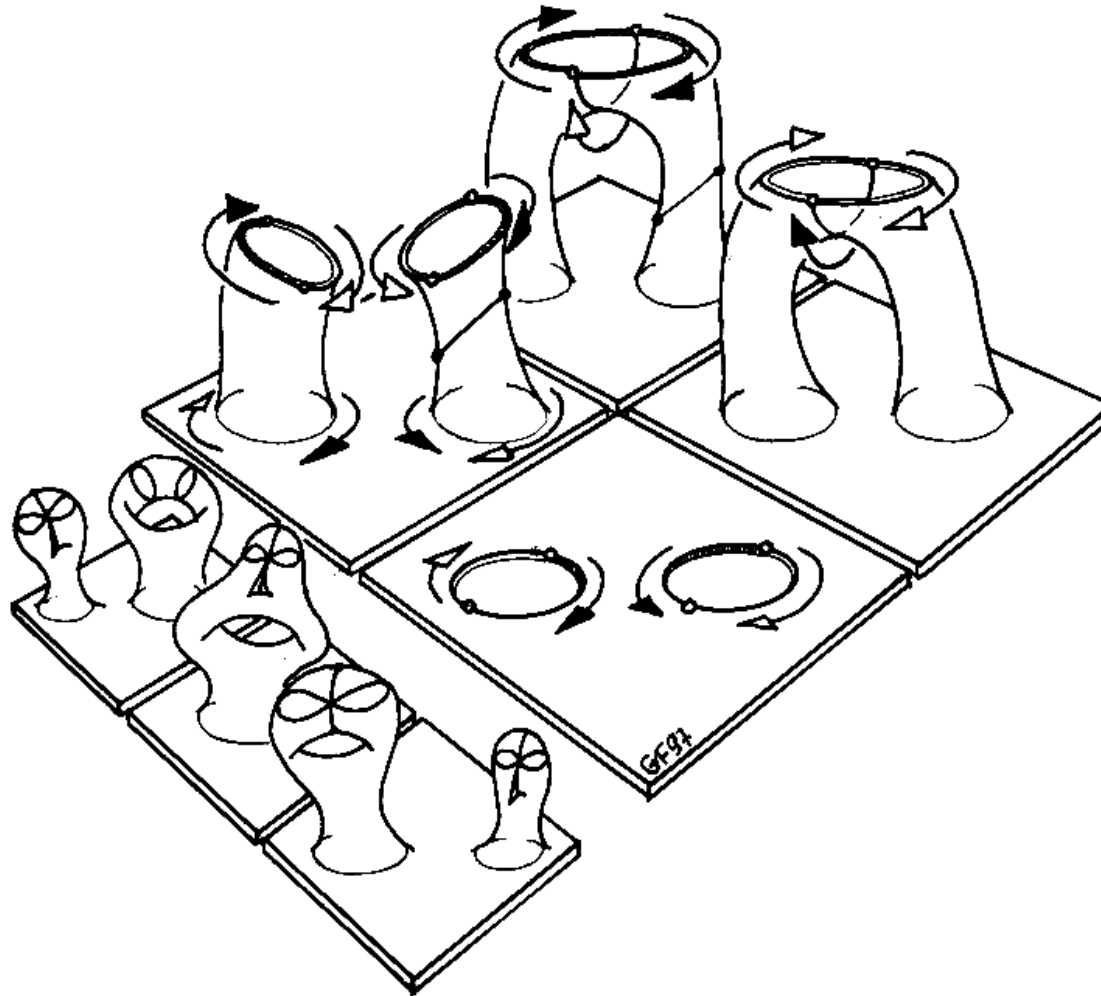
G.Francis and Jeff Weeks, "Conway's ZIP Proof, Amer Math Monthly, 1999

Dyck's Theorem, second draft



G. Francis and Jeff Weeks, "Conway's ZIP Proof, Amer Math Monthly, 1999

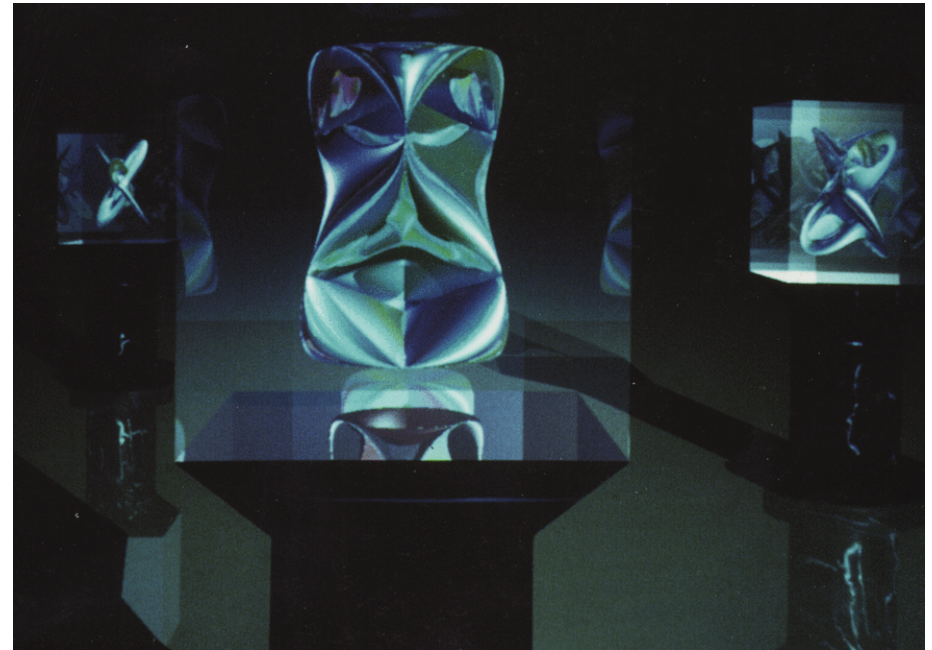
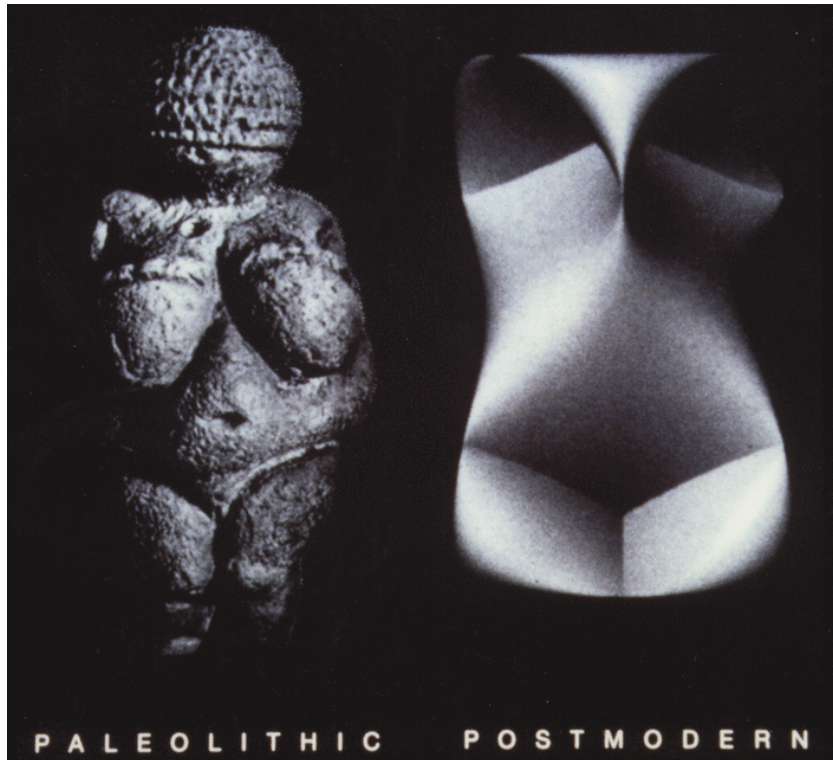
Dyck's Theorem: $3 \text{ xcaps} = 1 \text{ xcap} + 1 \text{ handle}$



G.Francis and Jeff Weeks, "Conway's ZIP Proof, Amer Math Monthly, 1999

Etruscan Venus

Cover Girls

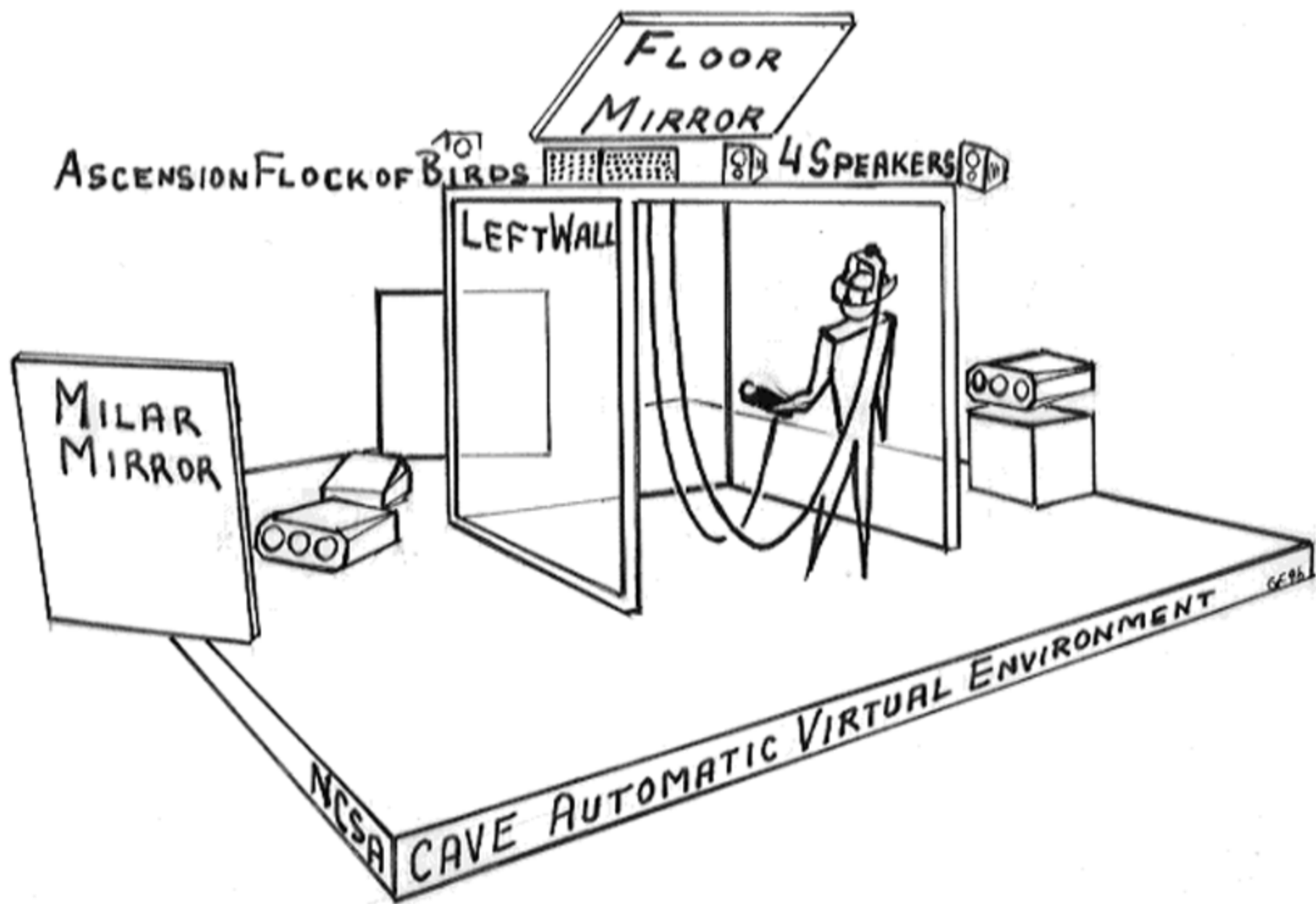


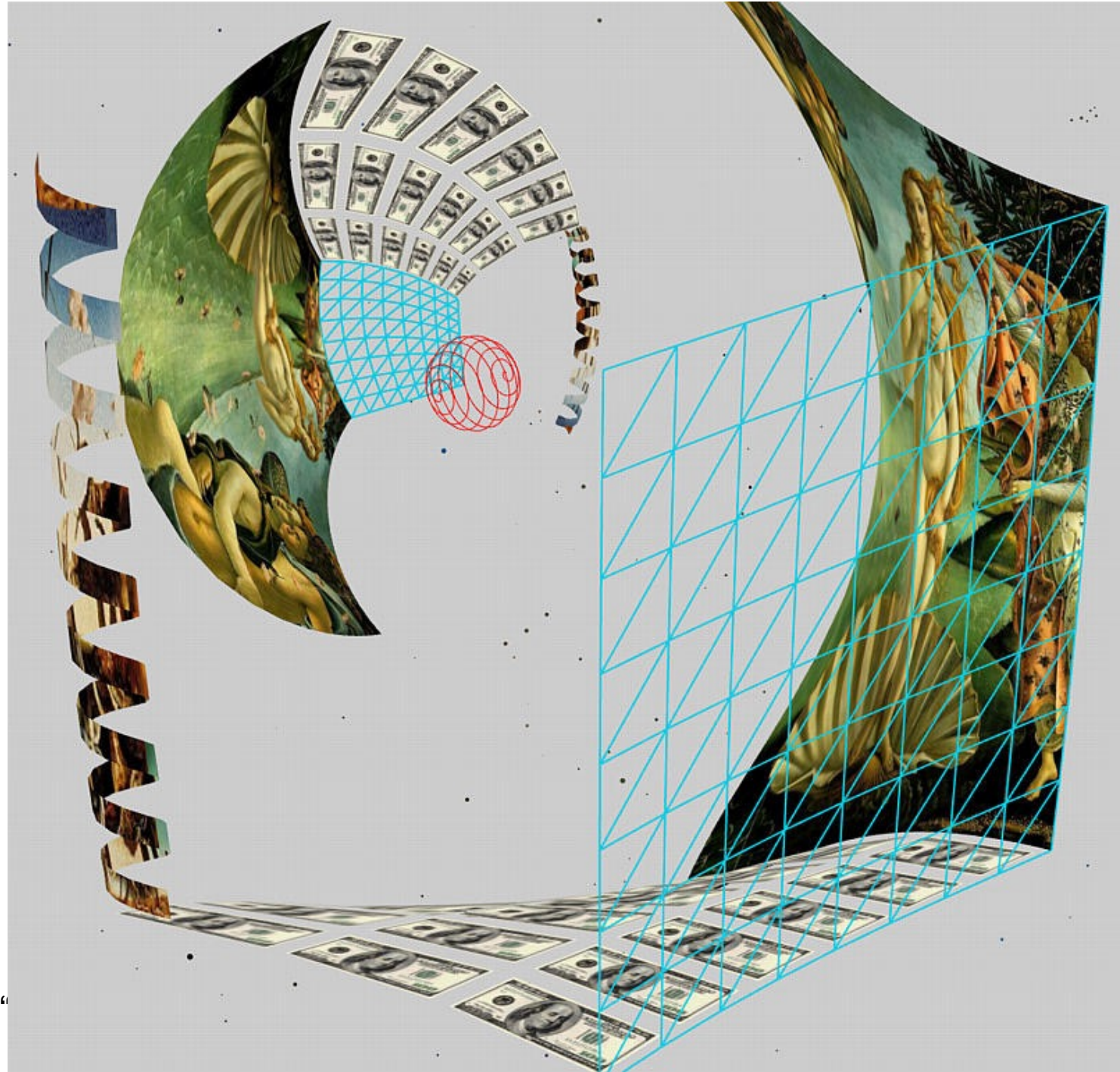
Crystalline Raytrace.

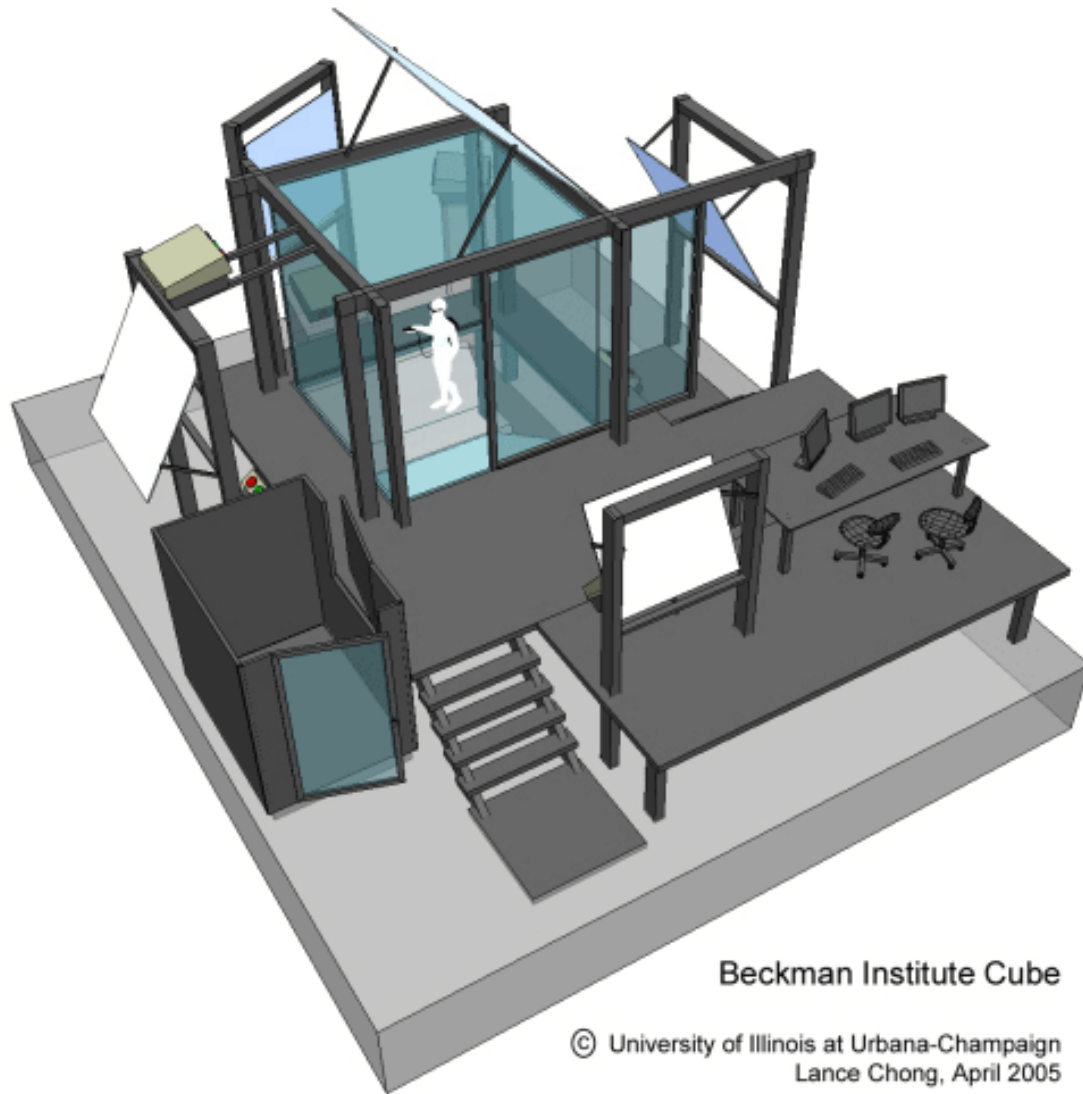
Donna Cox, George Francis, Ray Idaszak, NCSA ©1987

Apéry's Romboy Homotopy

A Real-time Interactive Computer Animation (RTICA)

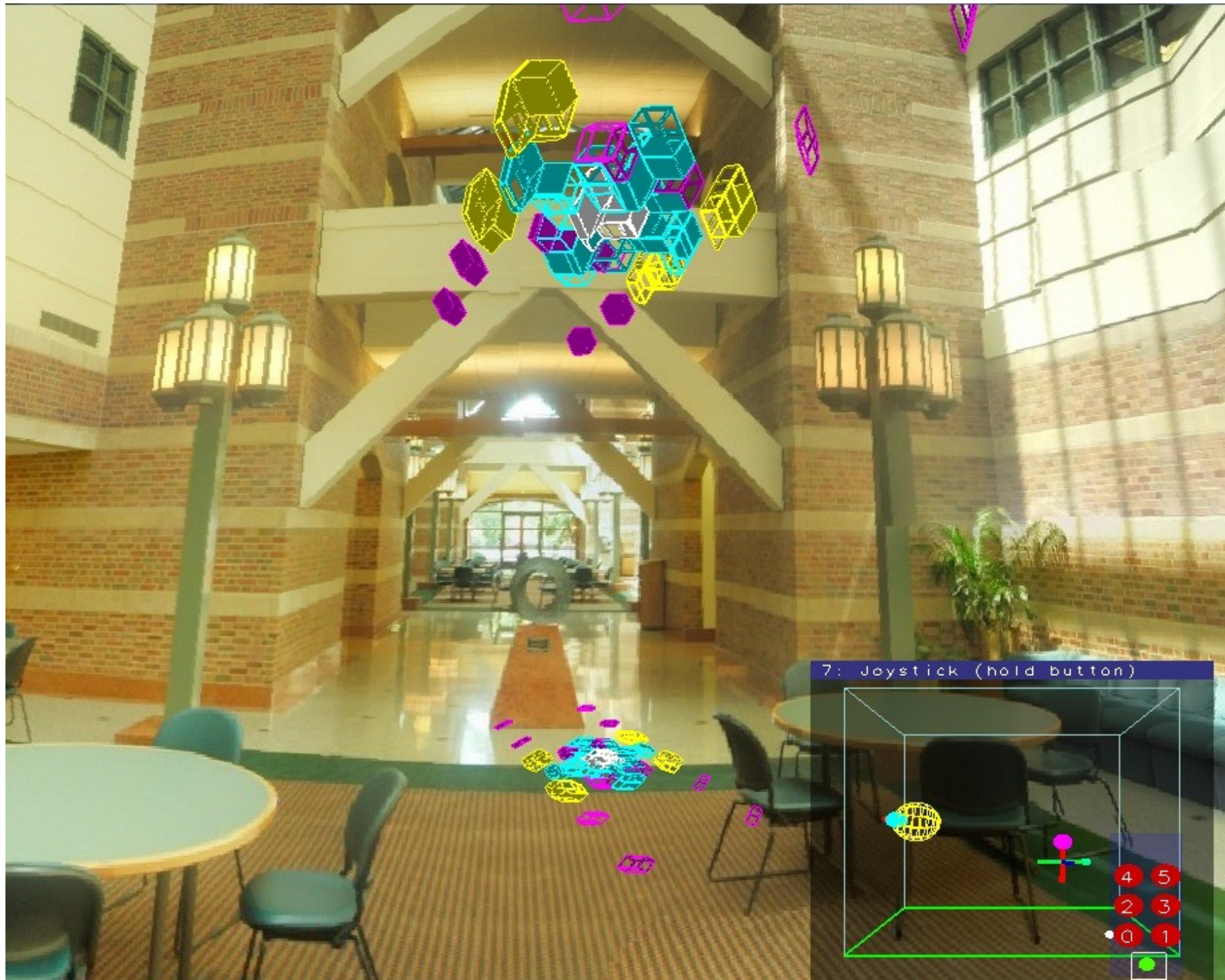






Beckman Institute Cube

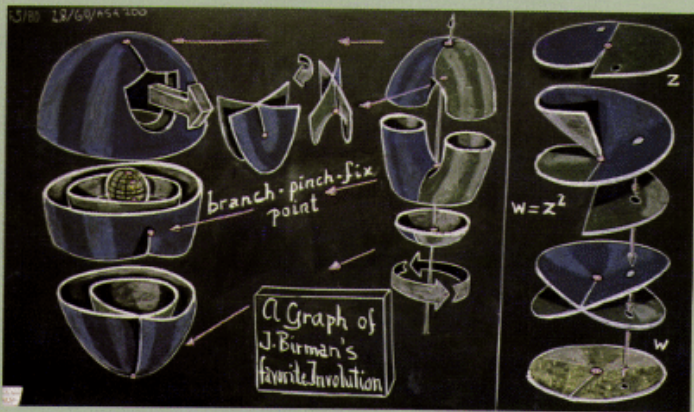
© University of Illinois at Urbana-Champaign
Lance Chong, April 2005



7: Joystick (hold button)



George K. Francis A Topological Picturebook



Springer-Verlag



Praise for George Francis's *A Topological Picturebook*:

Beats to Springer for reissuing this unique and beautiful book! It not only reminds the older generation of the pleasures of doing mathematics by hand, but also shows the new generation what "hands-on" really means.

—John Stillwell, University of San Francisco

A Topological Picturebook has taught a whole generation of mathematicians to draw, to see, and to think.

—Tony Robbin, artist and author of *Shadows of Reality: The Fourth Dimension in Relativity, Culture, and Modern Thought*

A Topological Picturebook is a visual feast for anyone concerned with mathematical images. Francis presents exquisite examples to build one's "visualization muscles." At the same time, he explains the underlying principles and design techniques for readers to create their own hand-drawn images.

—George W. Hart, Stony Brook University

The classic reference for how to present topological information visually, full of amazing hand-drawn pictures of complicated surfaces.

—John Sullivan, Technical University Berlin

In this collection of narrative gems and intriguing hand-drawn pictures, George Francis demonstrates the chicken-and-egg relationship, in mathematics, of theory and art. Since the book was first published, the case for pictures in mathematics has been won, and now it is time to reflect on their meaning. *A Topological Picturebook* remains indispensable.

—Marino Senechal, Smith College and co-editor of the *Mathematical Intelligencer*

A Topological Picturebook lets students see topology as the original discoverers conceived it: concrete and visual, free of the formalisms that burden conventional textbooks.

—Jef Weeks, author of *The Shape of Space*

To appreciate computer visualizations of challenging phenomena in geometry and topology, it is important to compare them with classical sketching and drawing techniques. George Francis is a master of both. This reprinting of the amazing book of topological images gives new generations of teachers and students the opportunity to profit from his insight and experience.

—Thomas F. Banchoff, Brown University



The cover shows 8 steps of a *Halfboard Drawing* (Chapter 1) of two positions for the *Selert surface* spanning a figure-8 knot (Chapter 8).

springer.com

Francis
A TOPOLOGICAL PICTUREBOOK

A TOPOLOGICAL PICTUREBOOK



George K. Francis



Springer

ISBN 4-431-71184-8

C3041 ¥2700E

定価(本体2,700円+税)



9784431711841



1923041027005



トポロジーの絵本

G.K.フランシス 著
笠原晴可 監訳 宮崎興二 訳



SPRINGER READINGS IN MATHEMATICS
シュプリンガー数学リーディングス 第4巻

A Topological Picturebook トポロジーの絵本

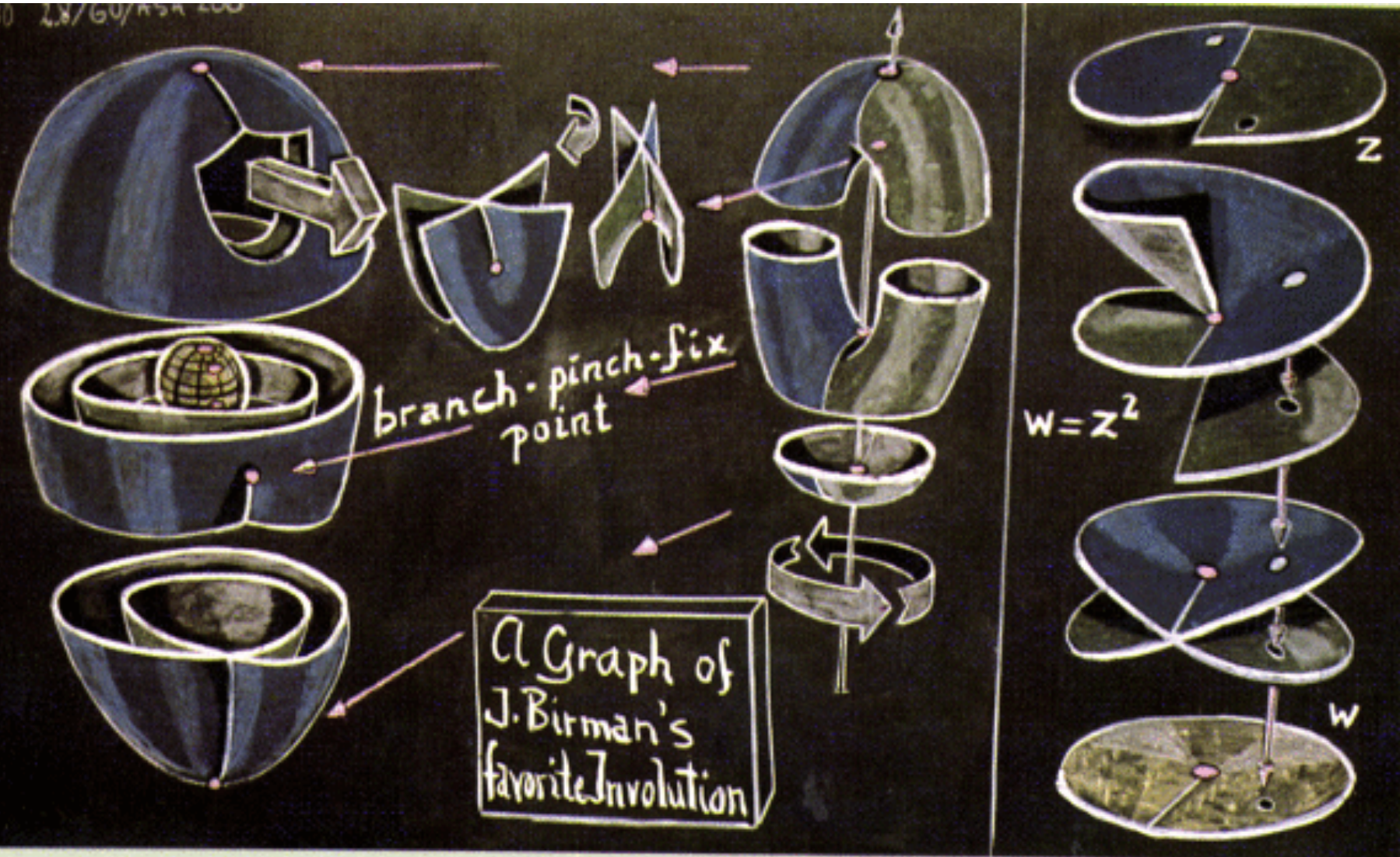
G.K.フランシス 著

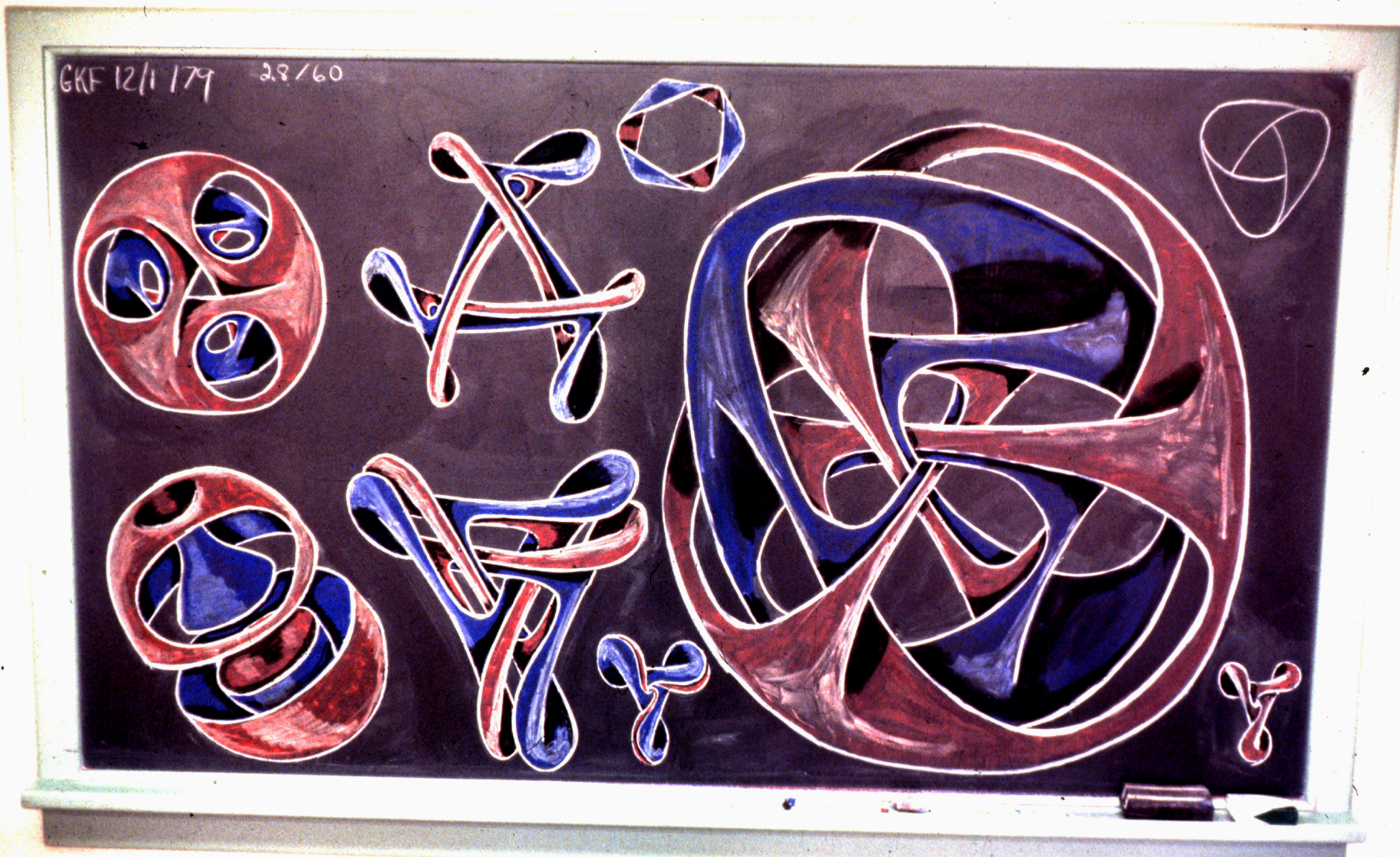
笠原晴可 監訳 宮崎興二 訳



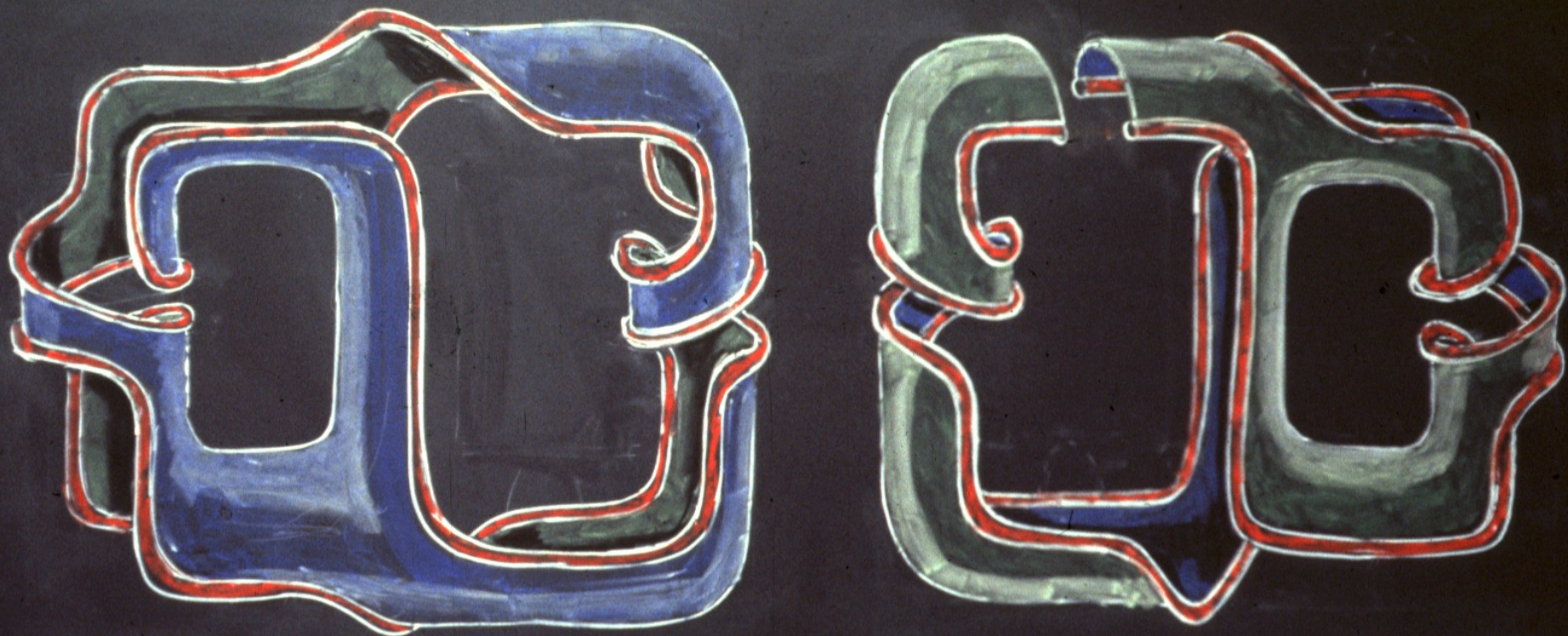
Springer
シュプリンガー・グループ

Joan Birman's Favorite Involution





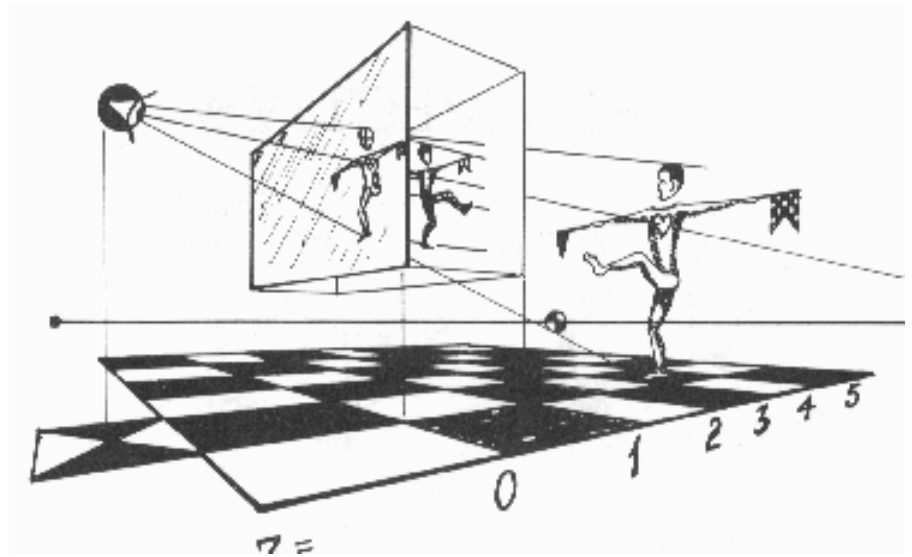
Unsymmetrical Triblague 1979



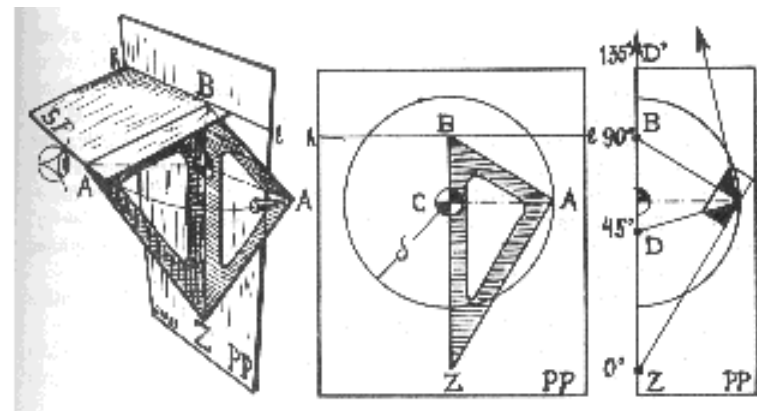
Isotopic surfaces spanning a Figure-8 knot

Linear Perspective

Renaissance
and
OpenGL

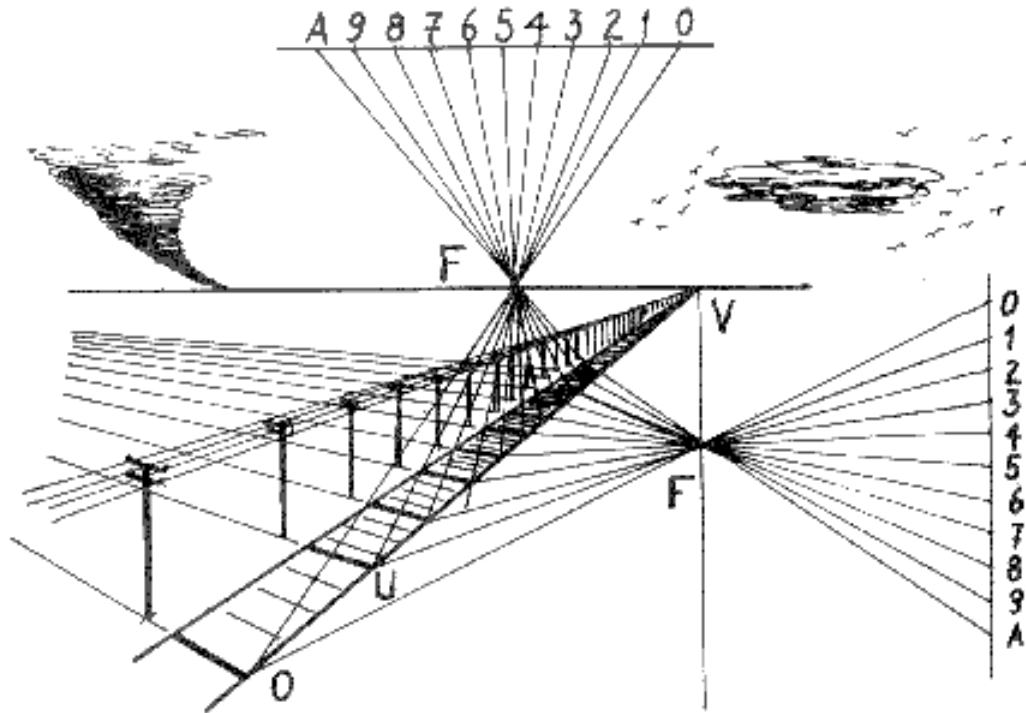


Horizon – Zenith
and the
Geometric Mean



from George K. Francis, "A Topological Picturebook", Springer-Verlag, 1987

Prairie Horizon



Crossratios are the rulers in projective geometry

Whitney Umbrella

original right
and
elaborated by
Lun Yi Tsai
below

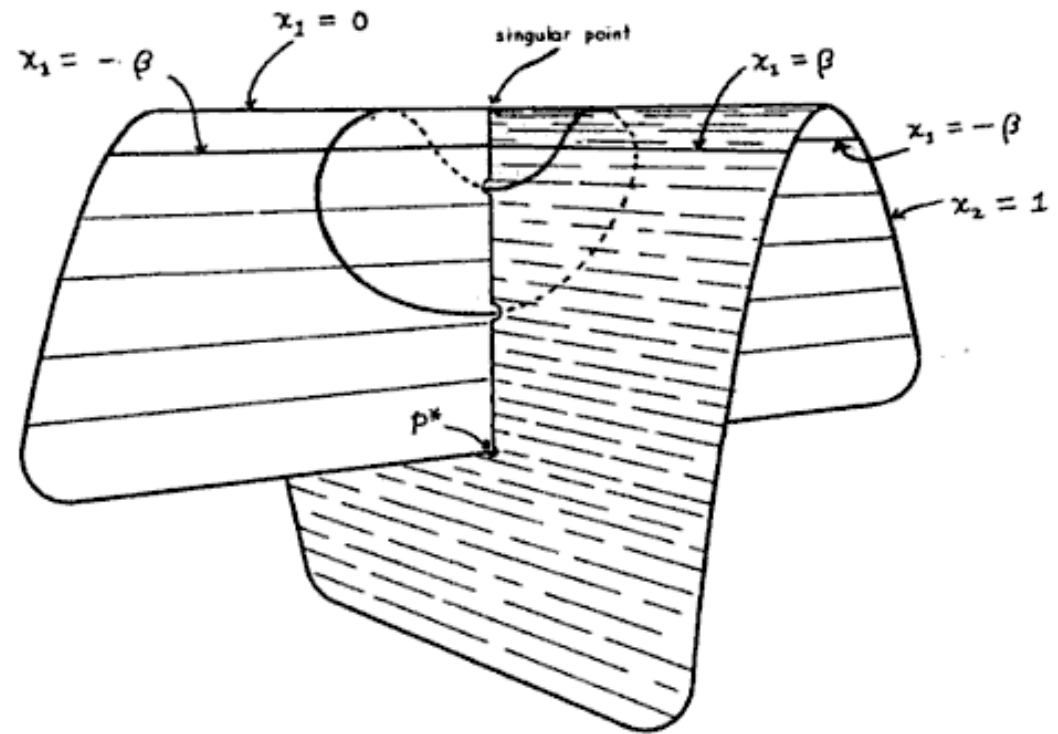
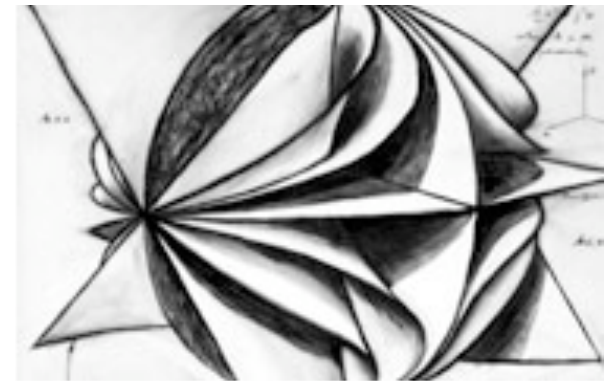


FIG. 1

4. Whitney Family

2008, charcoal and graphite on paper, 39 x 39 in

The Whitney Umbrella, a topological surface named for the American mathematician Hassler Whitney, can be thought of as a plane that is cut along a ray and glued back in such a way that it intersects with itself in three dimensions. This parameterization creates a stack of umbrellas sitting on their sides.



THE SINGULARITIES OF A SMOOTH n -MANIFOLD IN $(2n - 1)$ -SPACE*

BY HASSLER WHITNEY

(Received August 19, 1943)

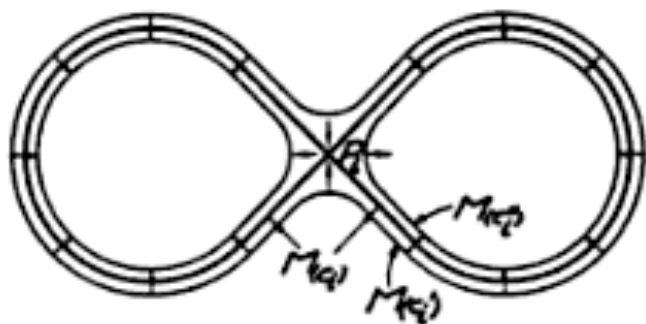


FIG. 1. Point p_i of type $1 \rightarrow 2$. Small arrows show direction of increase of f_i

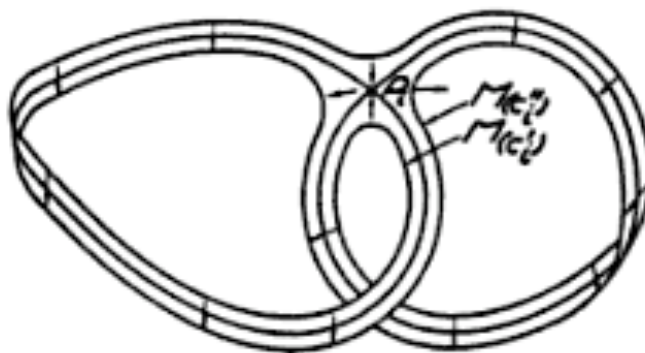


FIG. 2. Point p_i of type $1 \rightarrow 1$

Morse Theory

Boy's Surface
 (immersed projective plane)
 by sections

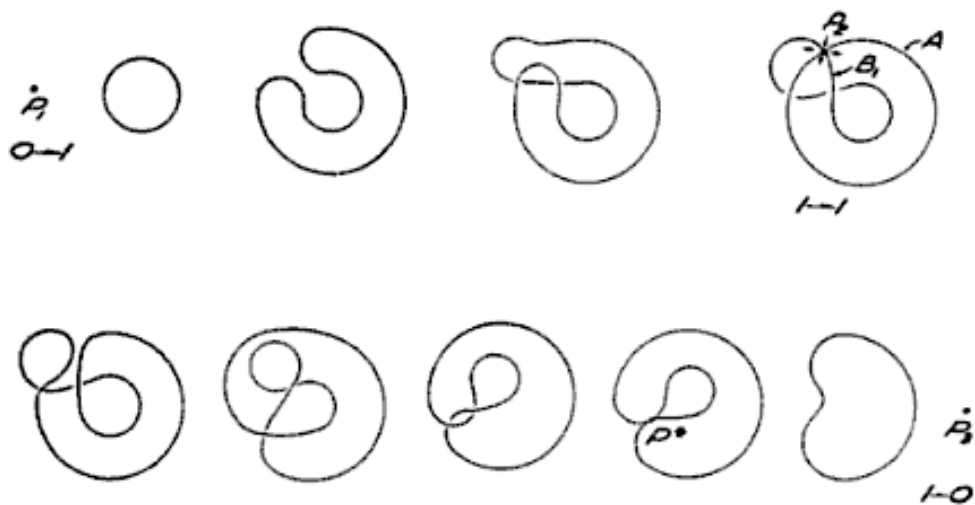


FIG. 6

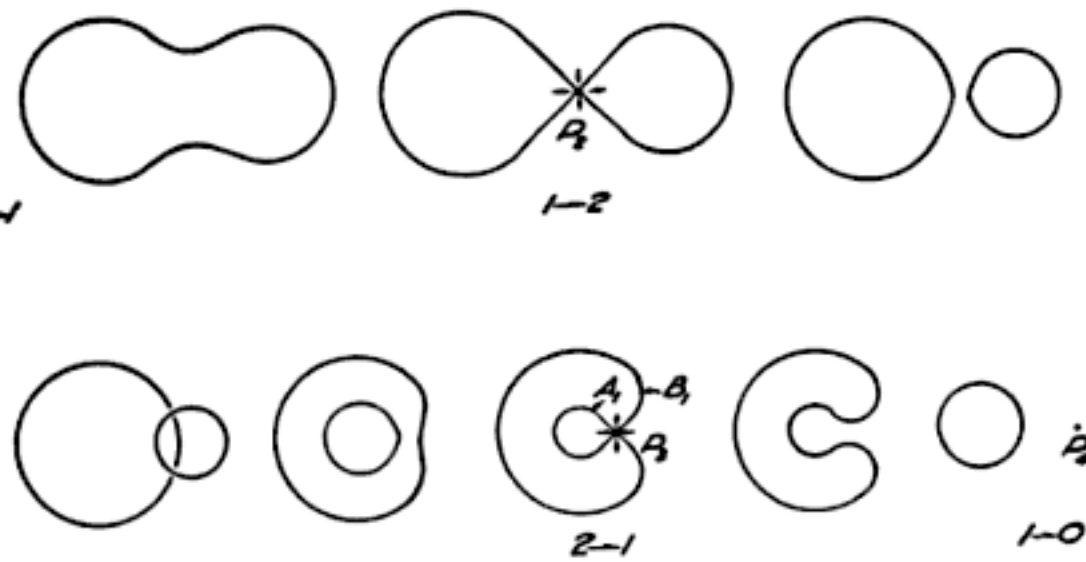


FIG. 3



Kleinbottle sections by Whitney

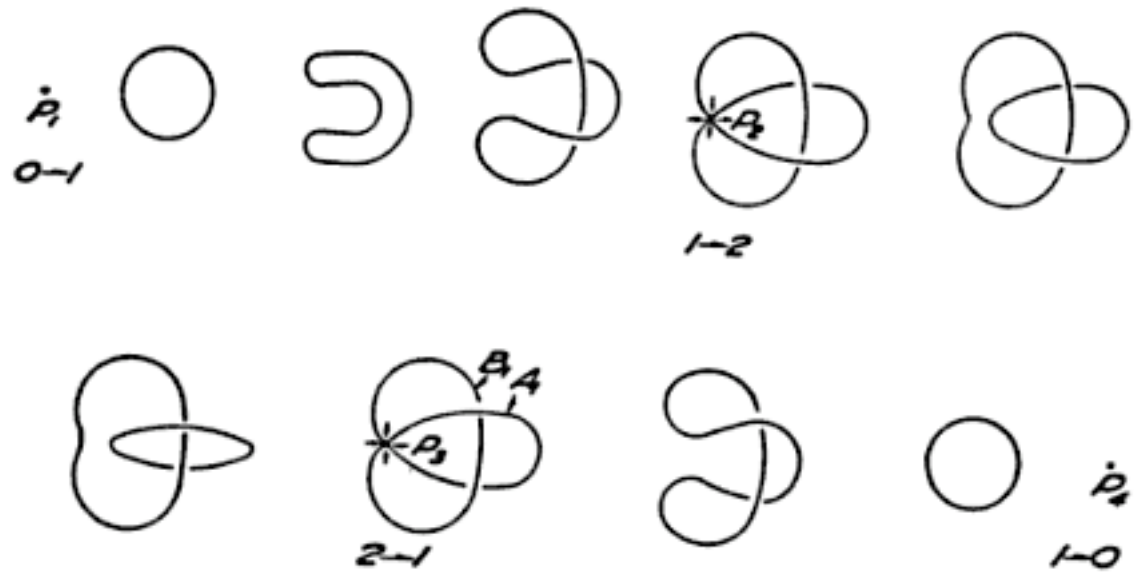
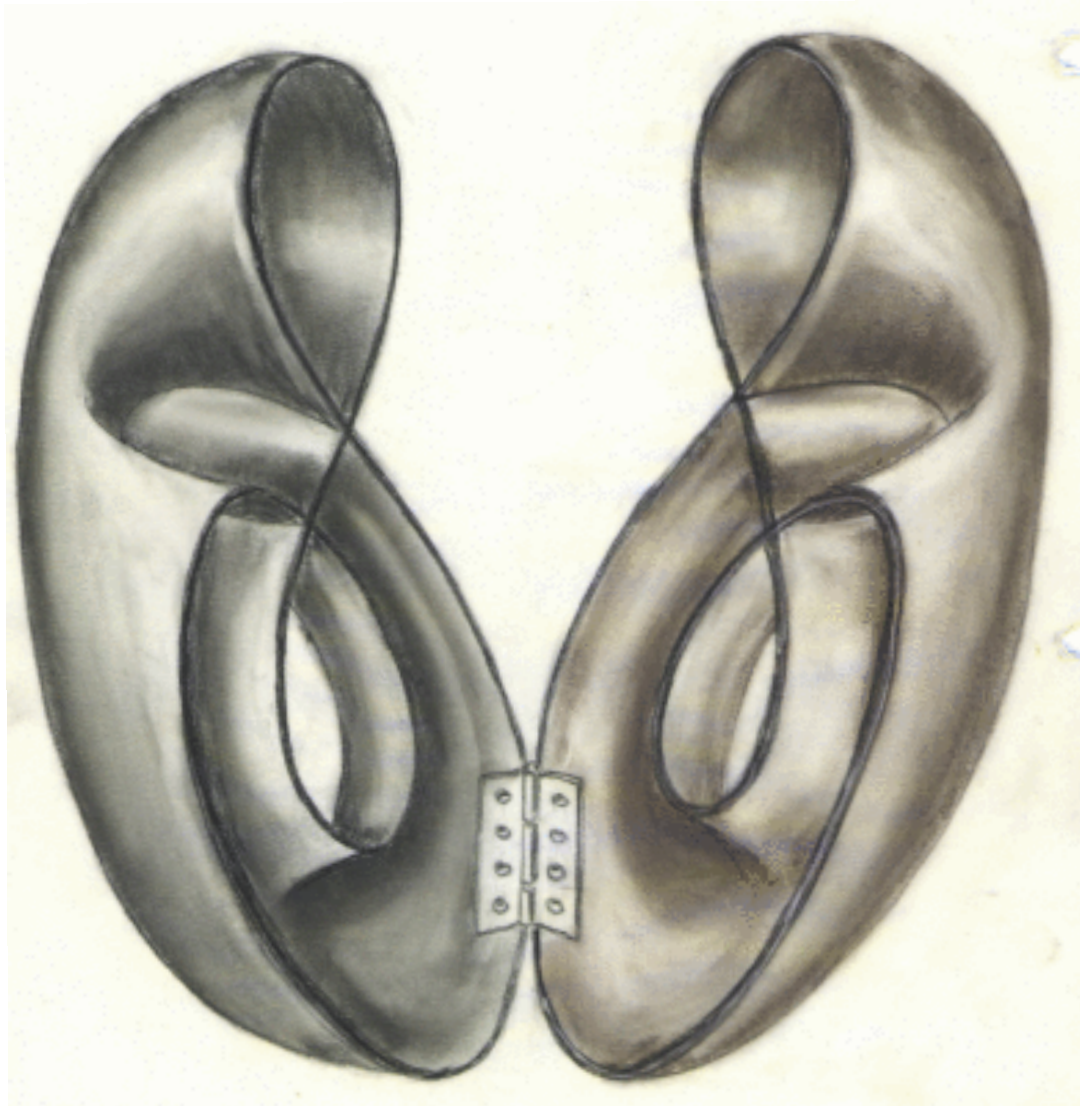


FIG. 4

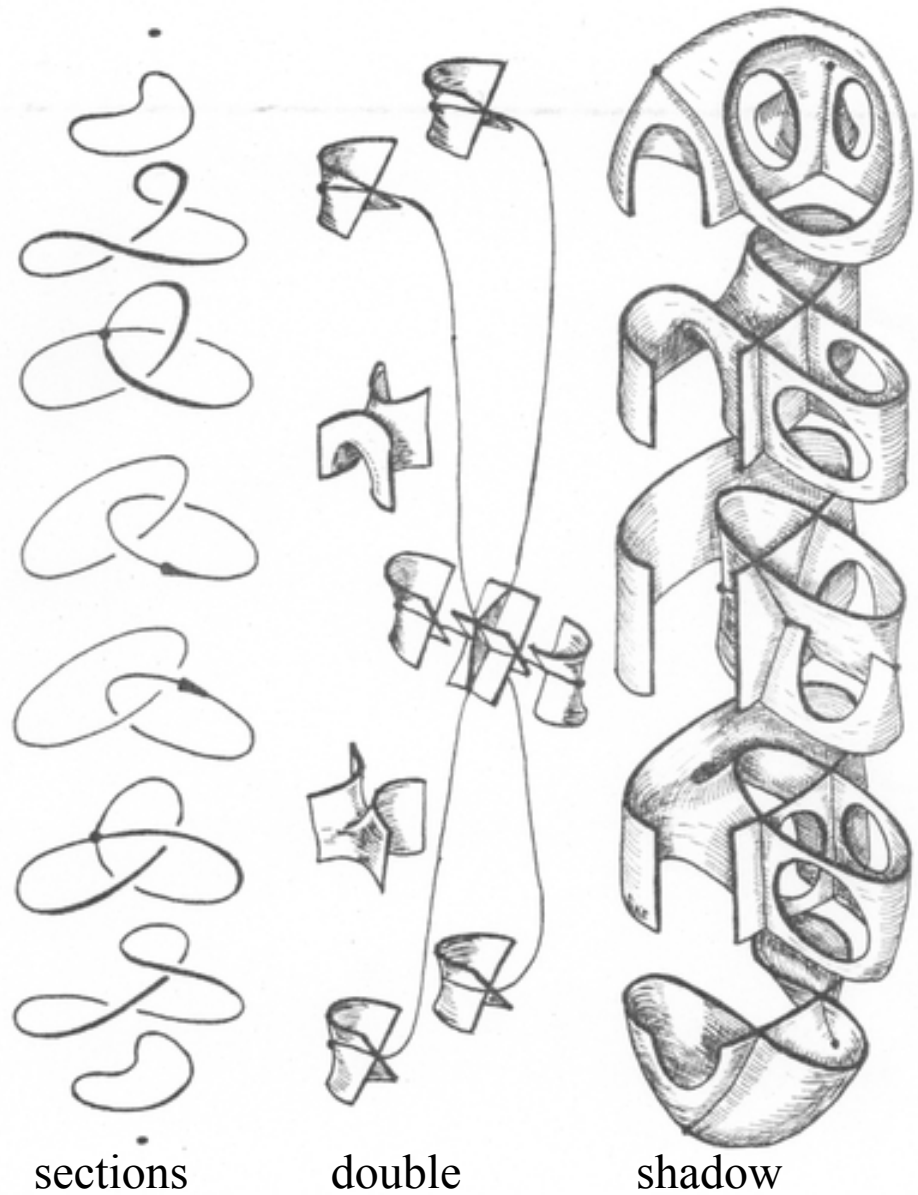
Hinged Kleinbottle



Pastel drawing of two immersed Möbius bands that form a Kleinbottle, 1983.

Whitney Bottle

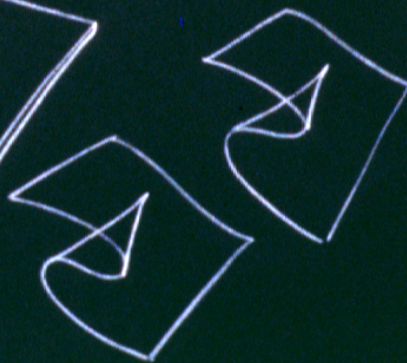
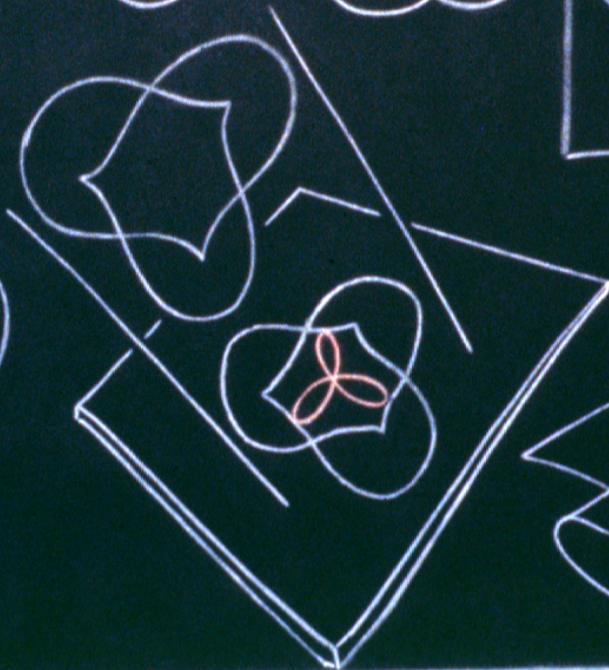
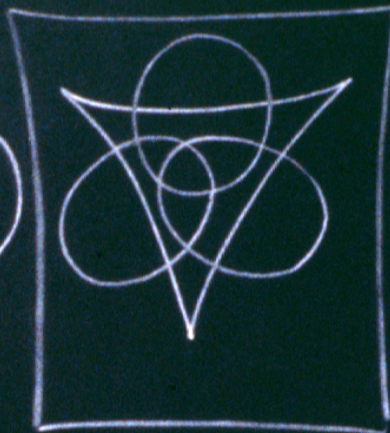
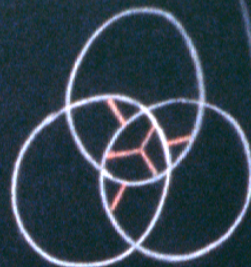
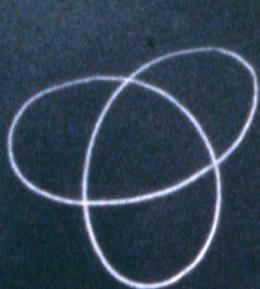
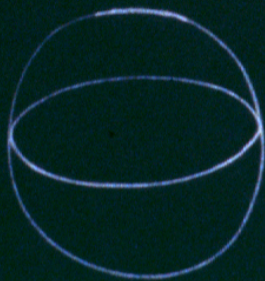
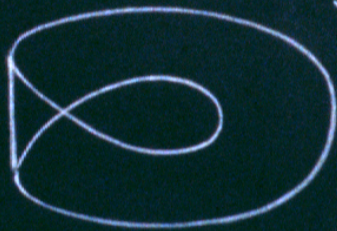
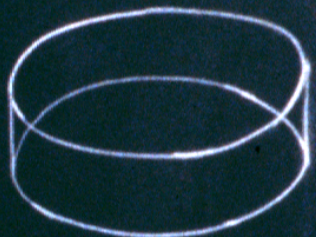
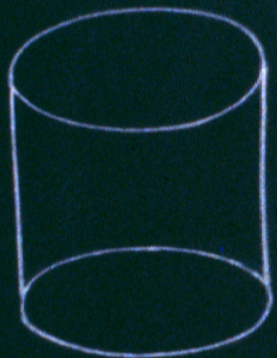
From
3-D sections
construct the
double locus,
& shadow of
a Kleinbottle
embedded in
4-Space



from George K. Francis, "A Topological Picturebook", Springer-Verlag, 1987

Chalk on Blackboard

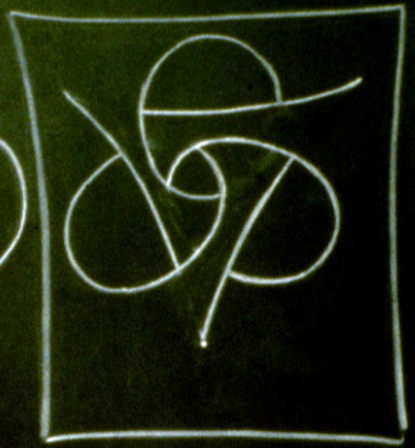
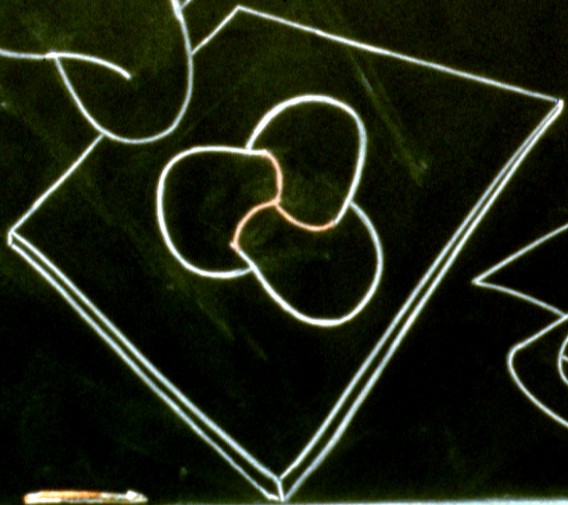
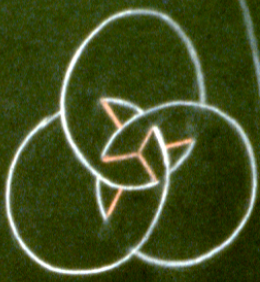
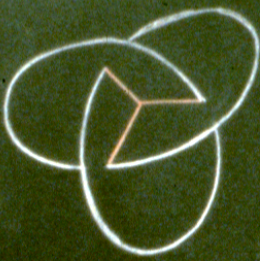
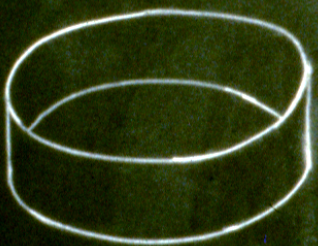
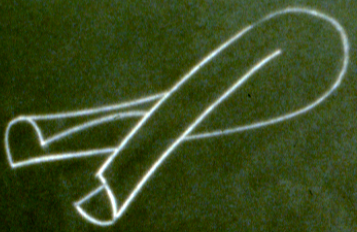
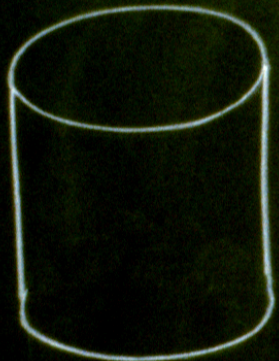
CHALK I



GKF 12/13/79

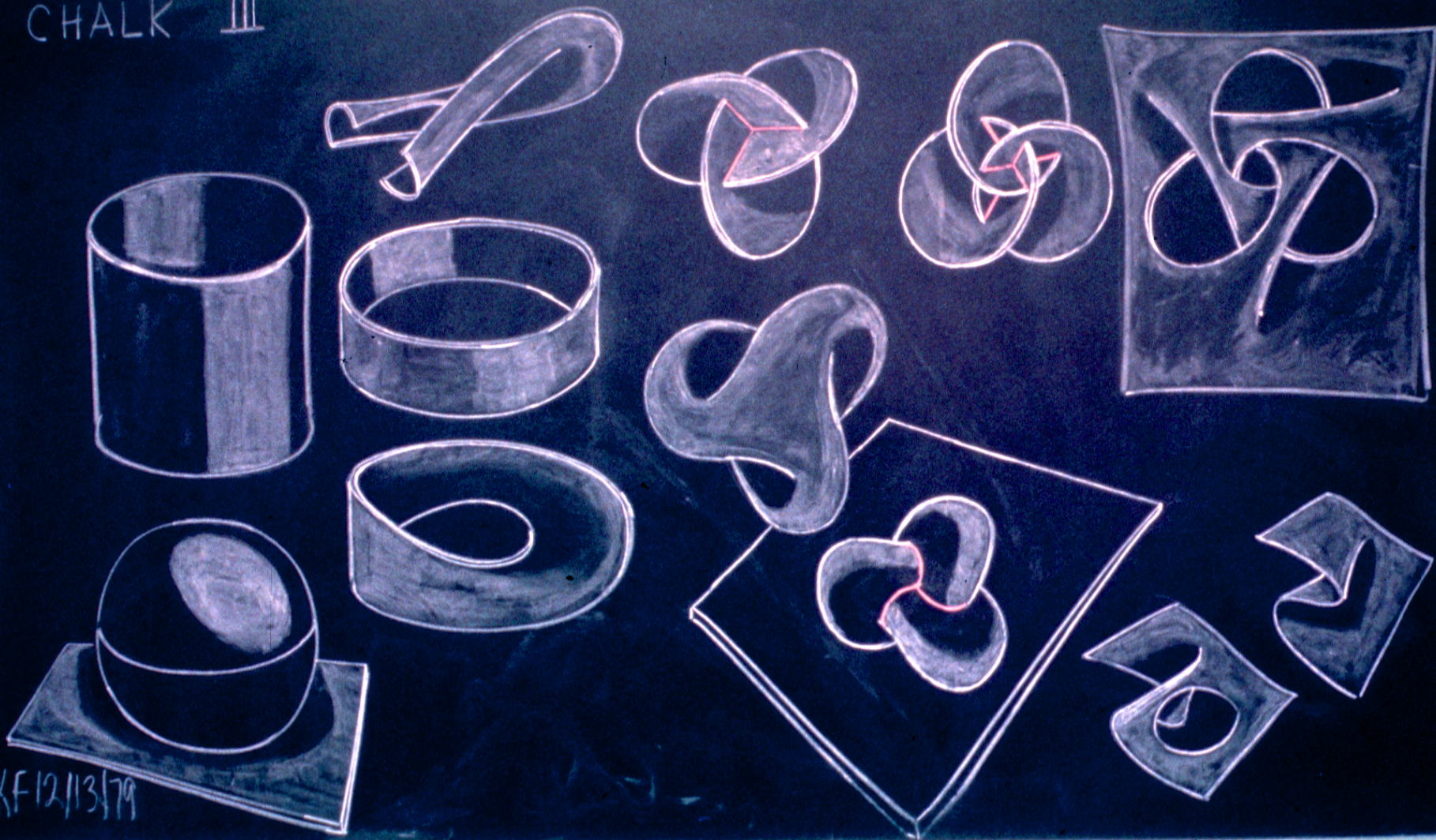


CHALK II



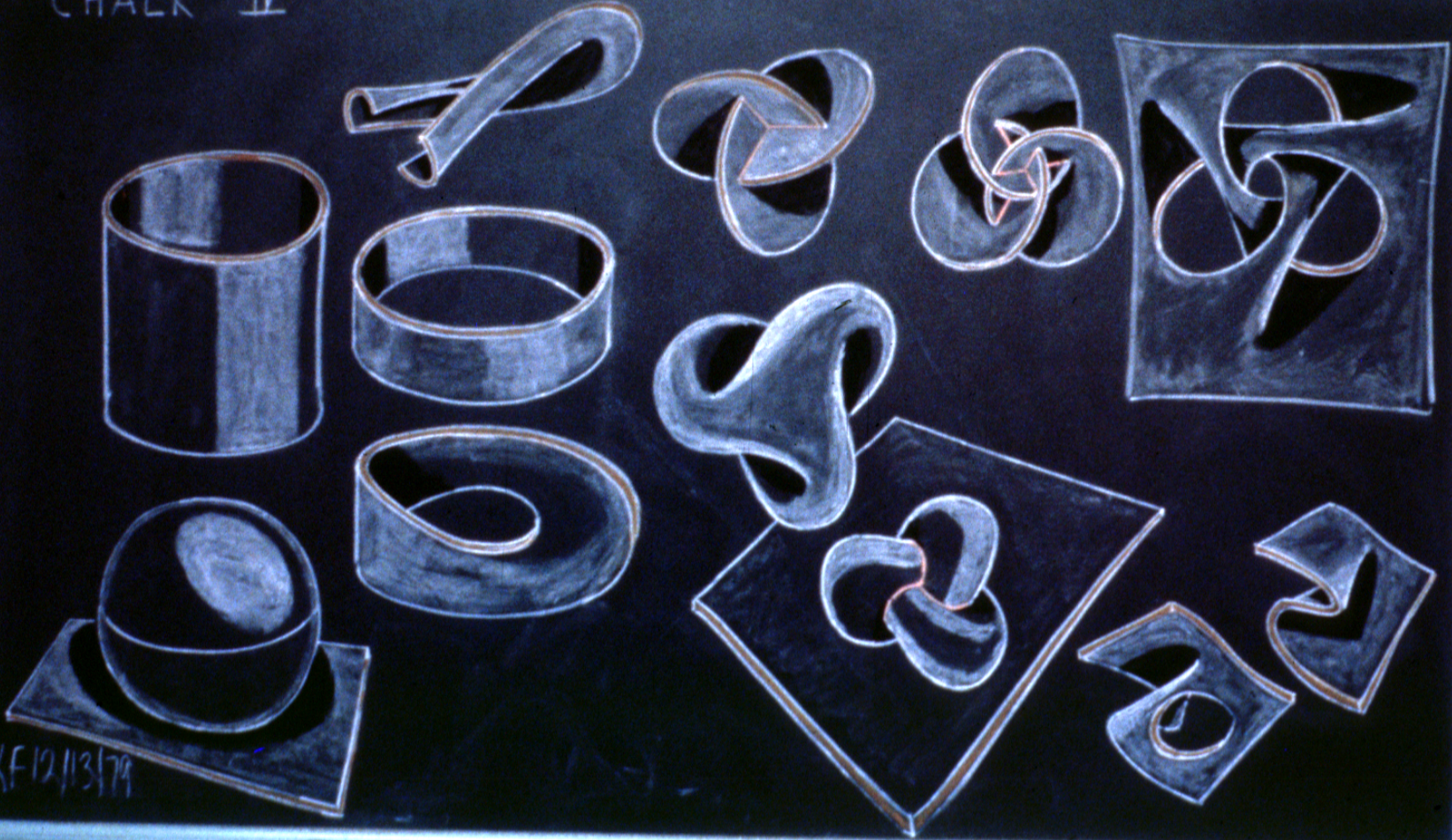
GKF 12/13/79

CHALK III



GKF 12/13/79

CHALK IV



GKF 12/13/79

Blackboard Dunce Cap

line drawing



from George K. Francis, "A Topological Picturebook", Springer-Verlag, 1987

Blackboard Dunce Cap

line drawing

windows & shadows



Blackboard Dunce Cap

line drawing

windows & shadows

coloring



from George K. Francis, "A Topological Picturebook", Springer-Verlag, 1987

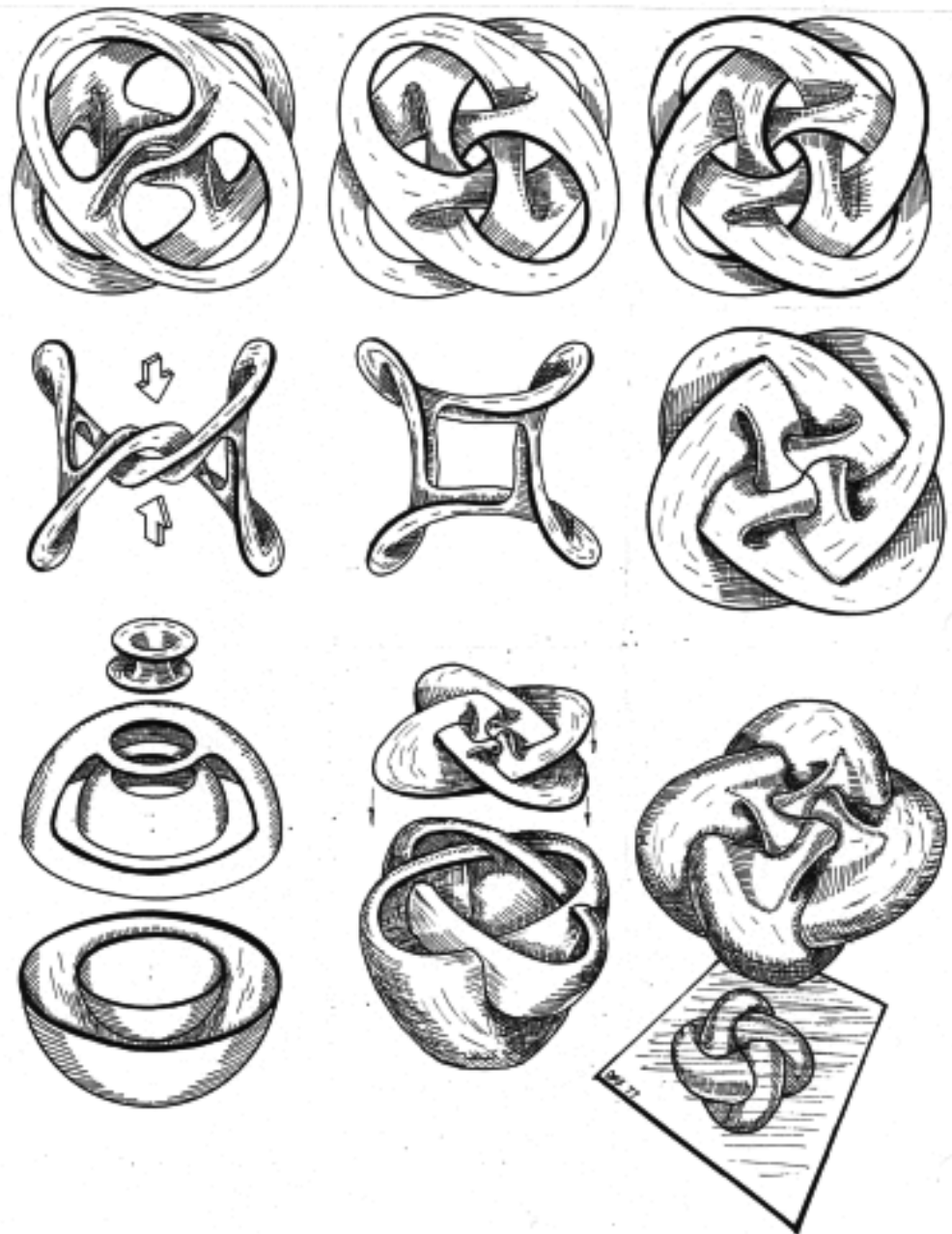
Blackboard Dunce Cap



from George K. Francis, "A Topological Picturebook", Springer-Verlag, 1987

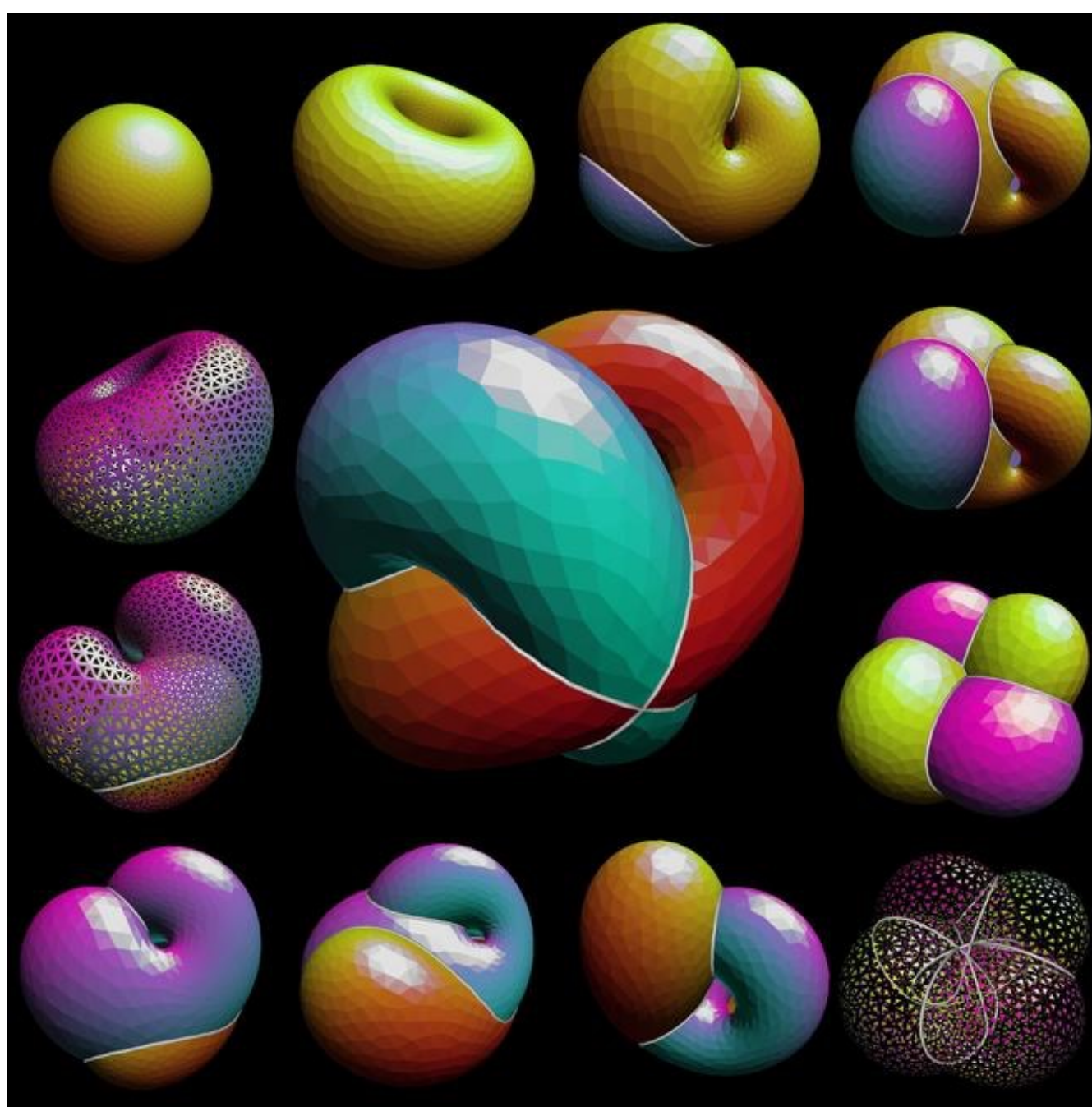
***John Dalbec's
Dunce Hat
RTICA***

Morin
Tableau
 $n=2$

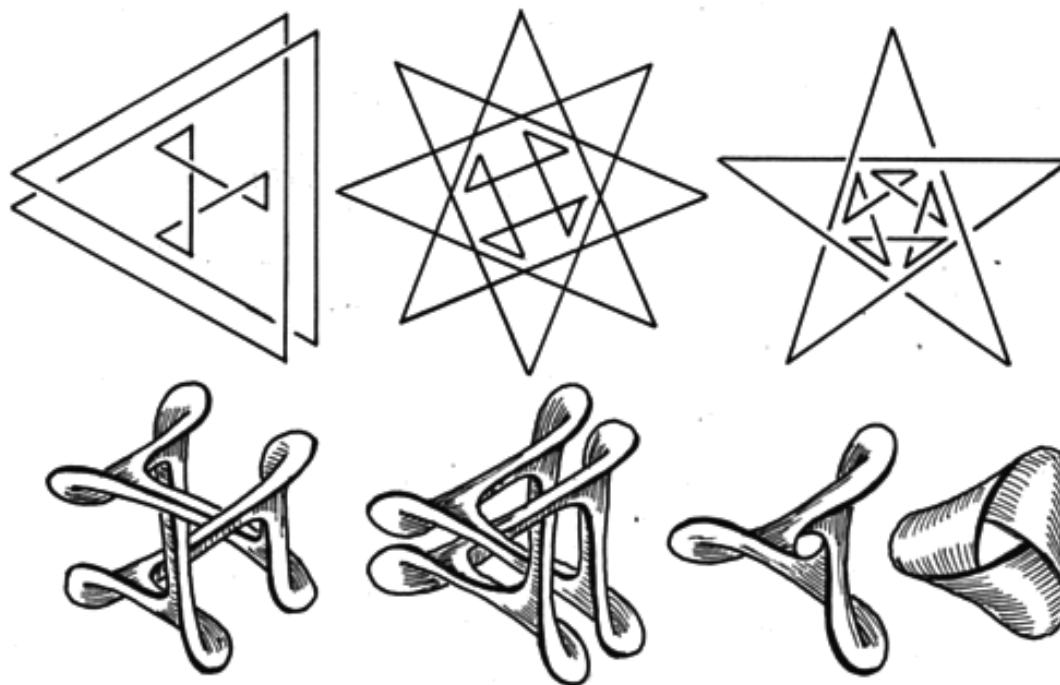
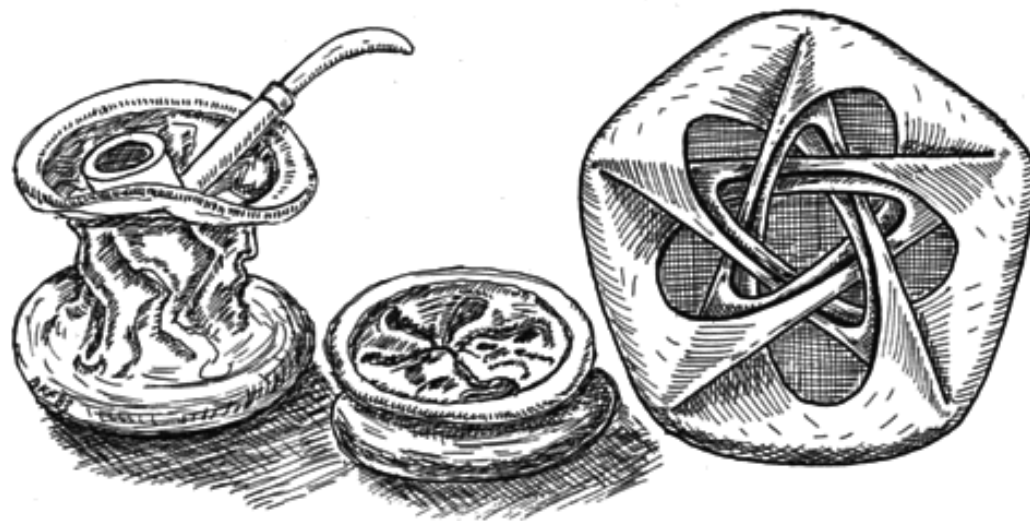


from George K. Francis, "A Topological Picturebook", Springer-Verlag, 1987

Morin
Montage
 $n=2$

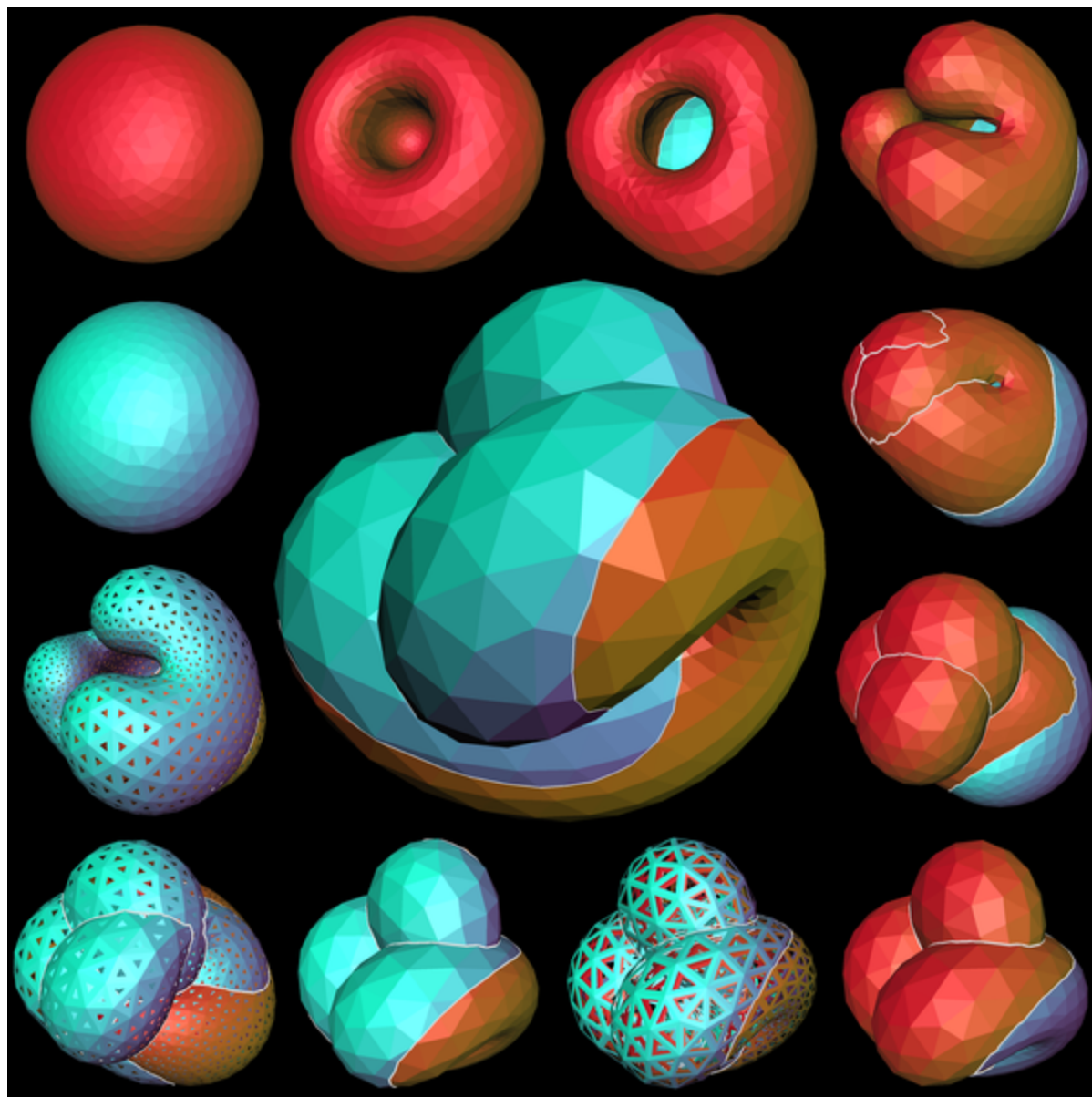


Boy
Tableau
 $n=3$



from George K. Francis, "A Topological Picturebook", Springer-Verlag, 1987

Boy
Montage
 $n=3$



*Bernard Morin “sees” his
Sphere Eversion*

Morin Eversion

1967

2000



Bernard Morin at Maubeuge, France, 20 September 2000



Bernard Morin

looking at
Stuart Dickson's stereolith models of
John Sullivan's *Minimax Eversion*,
Maubeuge, France, September, 2000

Dickson's Stereoliths



Five Morin-Apéry RTICAs

Quasicrystal Frameworks



COAST

Tony Robbin 1994

Center for Arts Sciences and
Technology at the

Danish Technical University

Erik Reitzel - engineer

RCM Precision - fabrication

Poul Ib Hendriksen - photos



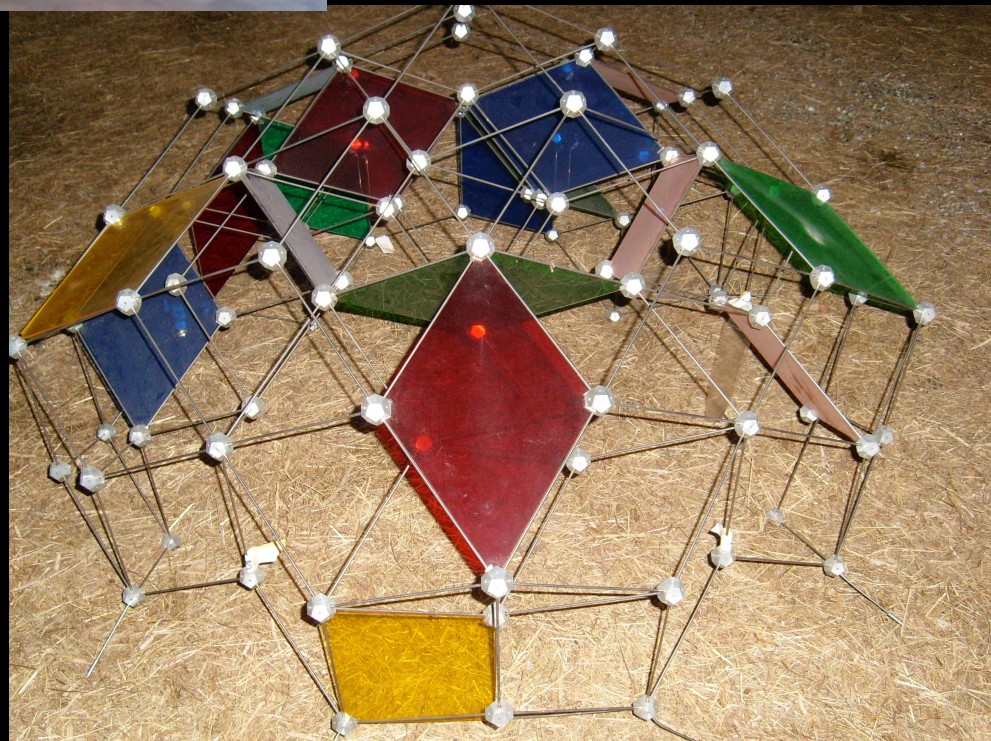




COAST was destroyed by
a misguided administrator



Tony Robbin, NY



*With thanks
to all in the*

*illiMath Collective
1977-2022*

Collaborators, co-authors, friends



Teachers:

PLATO 1977-1980:

Judy and Bruce Sherwood

Dept of Industrial Design:

Norm MacFarland, Ed Zagorski,
Vivian Faulkner-King

Student Assistants:

UIMATH.Applelab:1983-1994

Jim Bailey Ferrell Wheeler

Ted Emerson Cary Sandvig

REL/CAVE/grafiXlab:1988-2000

Ray Idaszak Glenn Chappell

Chris Hartman Alex Bourd

Ulises Cervantes-Pimentel

John Estabrook Matt Hall

Colleagues:

UIMATH.Applelab

Bob Illyes Graham Evans

NCSA, Urbana 1986-2000:

Donna Cox Carl Hoyer,
Bob Patterson Jeff Carpenter.

EVL, Chicago 1987-1998:

Dan Sandin Tom DeFanti Maxine

Brown Ellen Sandor Dana Plepys

Dave Pape Carolina Cruz-Neira

Geometry Center 1989-1997:

Pat Hanrahan, Charlie Gunn,

Stuart Levy, John Sullivan

Virtual Environments:

CAVE, CUBE, CANVAS (ISL) Ben Schaeffer, Jim Crowell, Camille Goudeseune,
Hank Kaczmariski

DiVE (Duke) Rachael Brady, David Zielinsky

Portal (TUB) Samy Khedem, John Sullivan, Steffen Weissman

Collaborators, co-authors, friends

Teachers:

PLATO 1977-1980:

Judy and Bruce Sherwood

Dept of Industrial Design:

Norm MacFarland, Ed Zagorski,

Vivian Faulkner-King

Student Assistants:

UIMATH.Applelab:1983-1994

Jim Bailey Ferrell Wheeler

Ted Emerson Cary Sandvig

REL/CAVE/grafiXlab:1988-2000

Ray Idaszak Glenn Chappell

Chris Hartman Alex Bourd

Ulises Cervantes-Pimentel

John Estabrook Matt Hall

Colleagues:

UIMATH.Applelab

Bob Illyes Graham Evans

NCSA, Urbana 1986-2000:

Donna Cox Carl Hoyer,

Bob Patterson Jeff Carpenter.

EVL, Chicago 1987-1998:

Dan Sandin Tom DeFanti Maxine

Brown Ellen Sandor Dana Plepys

Dave Pape Carolina Cruz-Neira

Geometry Center 1989-1997:

Pat Hanrahan, Charlie Gunn

Stuart Levy, John Sullivan

Virtual Environments:

CAVE, CUBE, CANVAS (ISL) Ben Schaeffer, Jim Crowell, Camille Goudeseune,

Hank Kaczmariski

DiVE (Duke) Rachael Brady, David Zielinsky

Portal (TUB) Samy Khedem, John Sullivan, Steffen Weissman

Research Experiences for Undergraduates

Audible Sketchpad (NCSA 1998-2000): Ande Croll , Jessica Jackson, Doug Nachand, Bob Pinta, Ben Shanbaum, Paul Whitaker, Matt Woodruff.

illiMath2001 (VIGRE): Ben Bernard, Ben Farmer, Mark Flider, Doug Nachand, Alison Ortony, Lorna Salaman, Ben Shanbaum, Robert Shuttleworth, Matt Woodruff.

illiMath2002(VIGRE): Amit Chatwani, Ben Farmer, Abdul Hamide, Brad Henry, Wendy Hubbard, Yana Malysheva.

PyCube2004 (Math Dept): William Baker, Blair Flicker, Emily Gunawan, Greg Stanton, Brett Witt.

illiMath2006 (REUsite): Dave Bergman, Nicholas Duchnowski, Emily Echevarria, Matt Gregory, Paul Prue, Chris Rainey, Mimi Tsuruga, Abby Watt.

illiMath2008 (REUsite): Chase Boren, Will Davis, Abdul Dakkak, Geoff Ehrman, Lisa Hickock, Sam Ostling, John Pacey, Katie Poon, Liz Rogers.

illiMath2010 (REUsite): Chris Bisom, Ian Markwood, Dan Rajchwald, Justin Schirle

Associated Mentors: Robert Acar (Puerto Rico), Peter Brinkmann (CCNY), Ulises Cervantes-Pimentel (WRI), Elizabeth Denne (Harvard), Abdul Dakkak (WRI), Paul McCreary (Evergreen), Mike Pelsmajer (IIT), Karen Shuman (Grinnell), Rose Marshack (ISU), Tony Robbin (NY), Jeff Weeks (NY).

Projects in the Illinois Geometry Lab

Stability of Quasicrystal Frameworks

Spring 2013: Alex Burnley, Chong Han

Fall 2023: Keran Huang, Natchiket Joshi, Jonathan McGreal

Spring 2014: Zachary Miksis, Daniel Pugliese, Joseph Zeller

Spring 2017:

The 3D project: Yijing Chen, Arturo Guerrero, Sasha Lamtyugina, Yi “Lisa” Li

The 2D project: Pranav Bhardwaj, Manting Huang, Tejo Nutalapati, Sung Jib Kim

Graduate Mentor: Eliana Duarte

Computer Visualization in Experimental Mathematics

Fall 2019: Joshua Holder, Xiaomin Li, Zhuoyun “Doris” Wang, Jinlin Xu

Graduate Mentors: Daniel Carmody, Karthik Vasu

visit

<http://new.math.uiuc.edu>

<http://new.math.uiuc.edu/puppetshow>

and

email me

gfrancis@illinois.edu