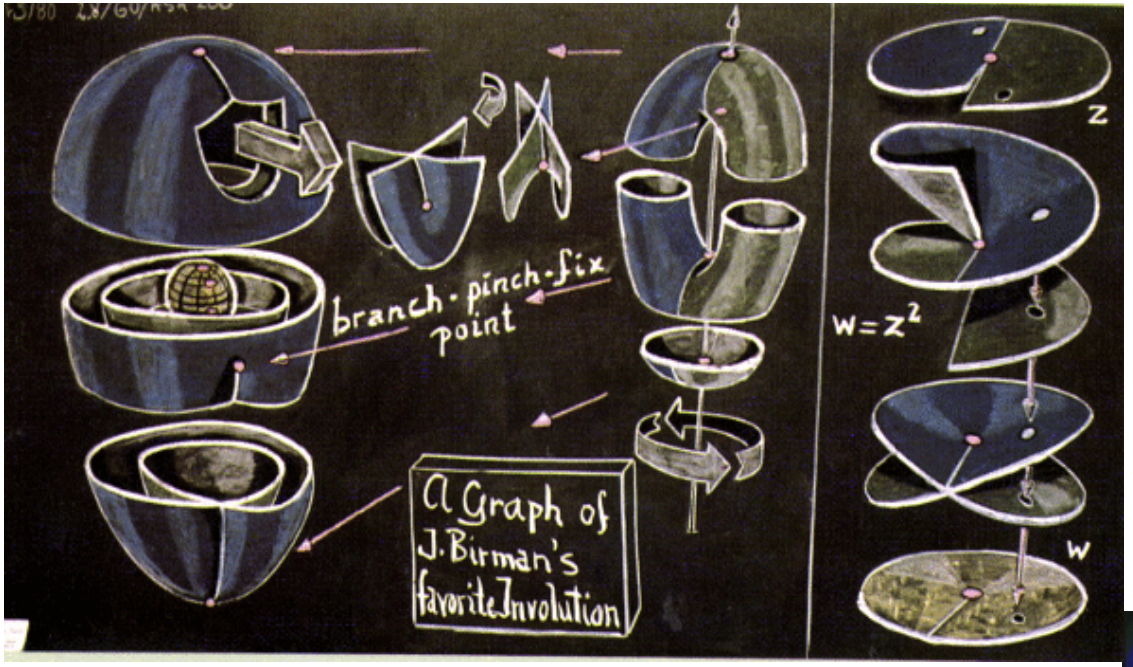
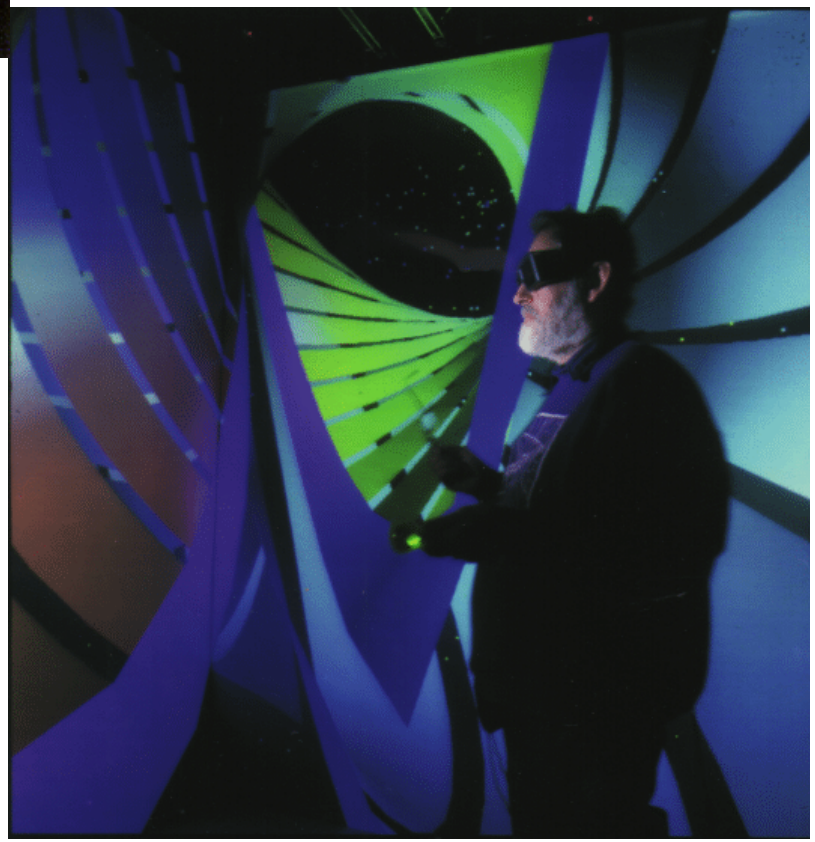
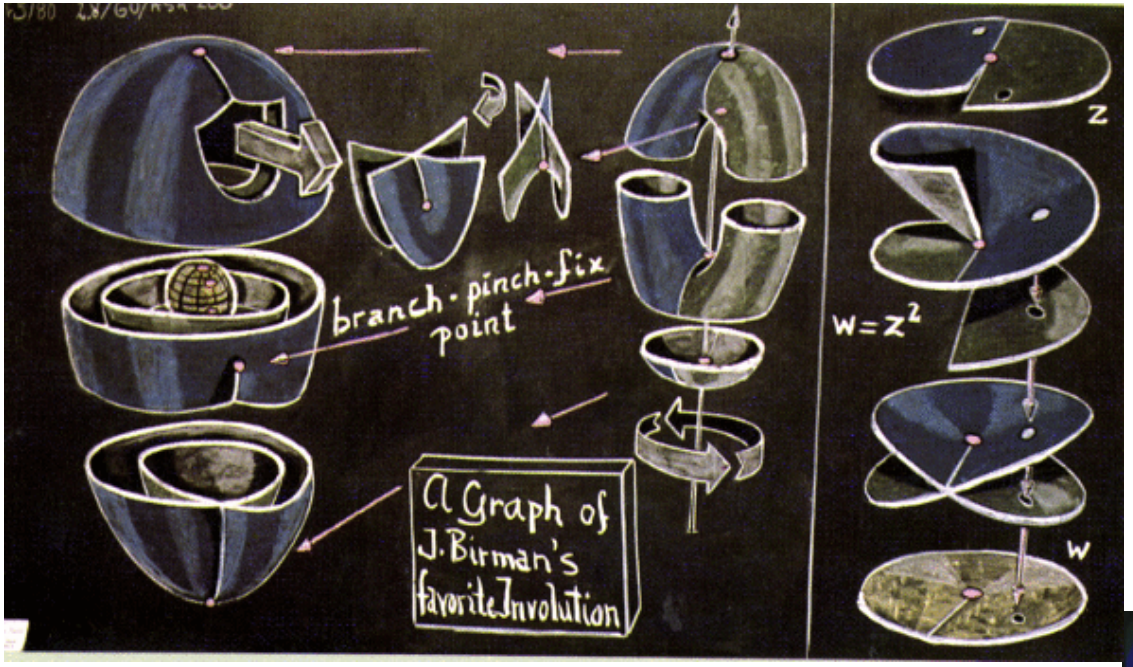


MathViz: is that any way to spend one's time?

George K. Francis
University of Illinois

Tucson Colloquium
24 March 2011



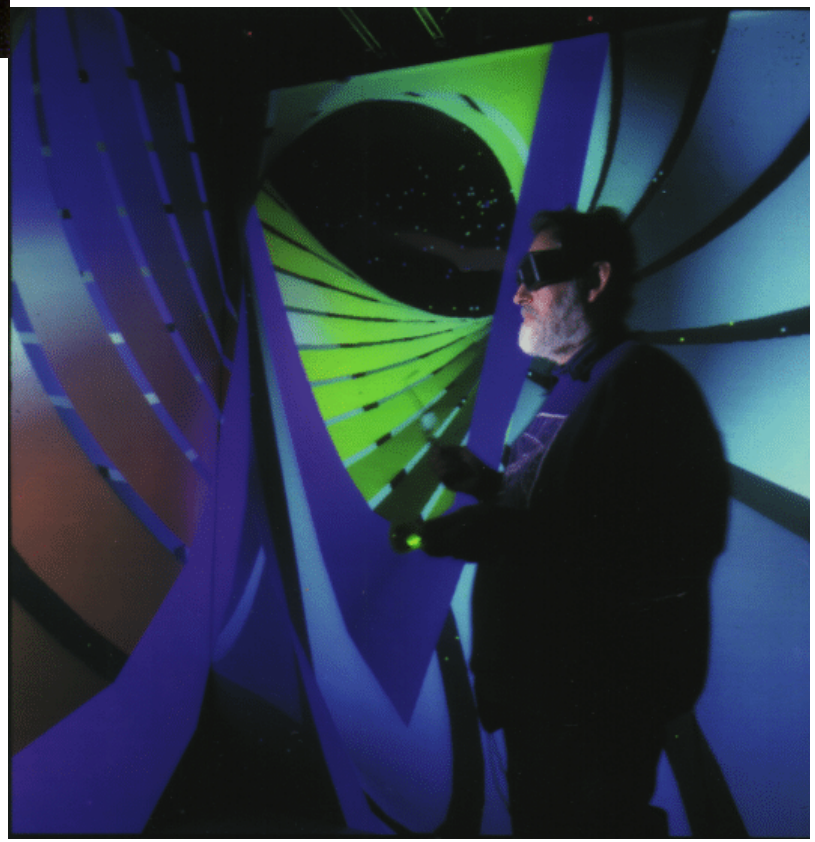


MathViz: is that any way to spend one's time?

George K. Francis

Math 499 – 6may15

University of Illinois



Program

The Preface: From Pythagoras to Bourbaki
The Optiverse Video:
A real-time interactive computer animation (RTICA)
Collaborators: The illiMath Collective
The Talk: The Topological Picturebook
Alberti's Veil (linear perspective)
OpenGL Pipeline
Klein Bottles
Blackboard Drawings
Duncehat and Trefoil
Morin's Sphere Eversion
3D Quasicrystals
The Encore: The Optiverse (RTICA)

Program

The Preface: Pythagoras
The Video: The Optiverse
The Talk: Collaborators
The Topological Picturebook
Alberti's Veil (Perspective)
OpenGL Pipeline
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Blackboard Drawing
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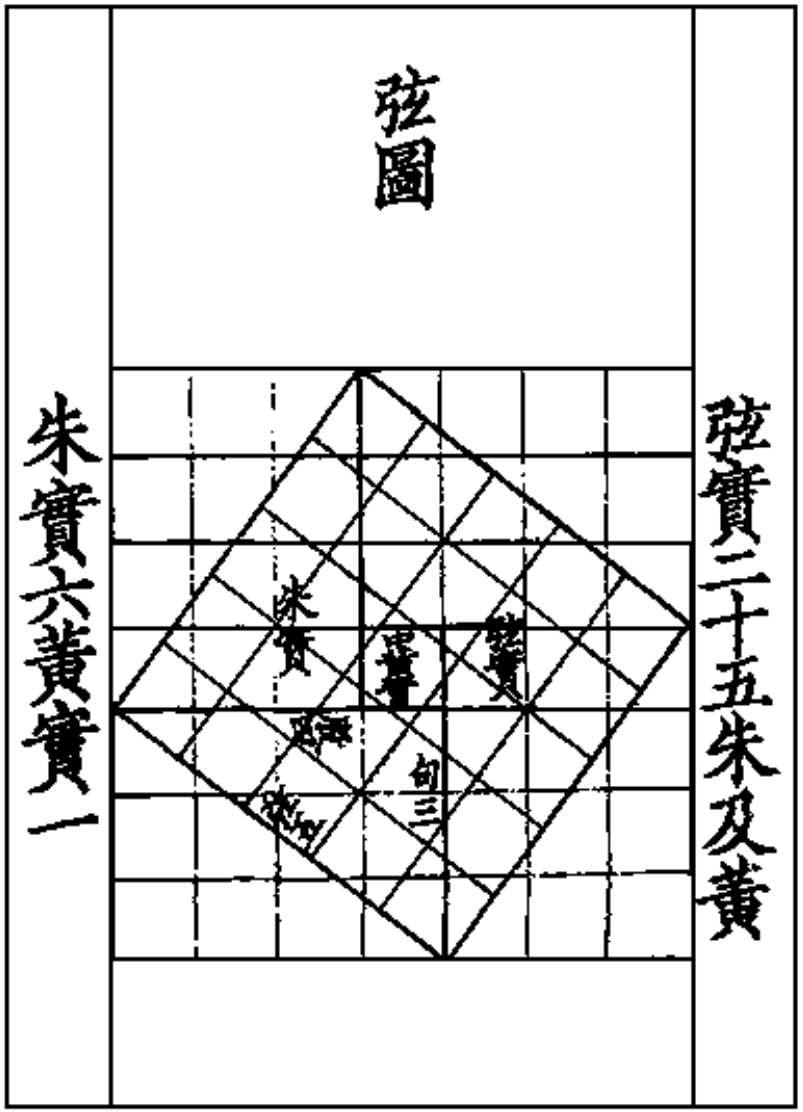
Morin's Sphere Eversion
3D Quasicrystals
Some Obligatory Theory
The Encore:
The Optiverse (RTICA)



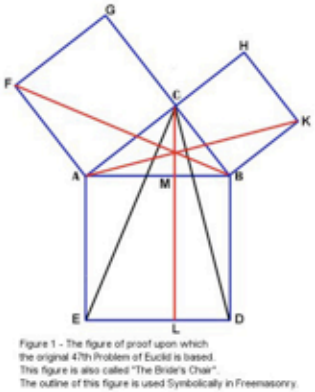
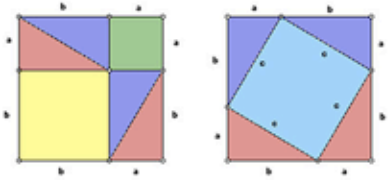
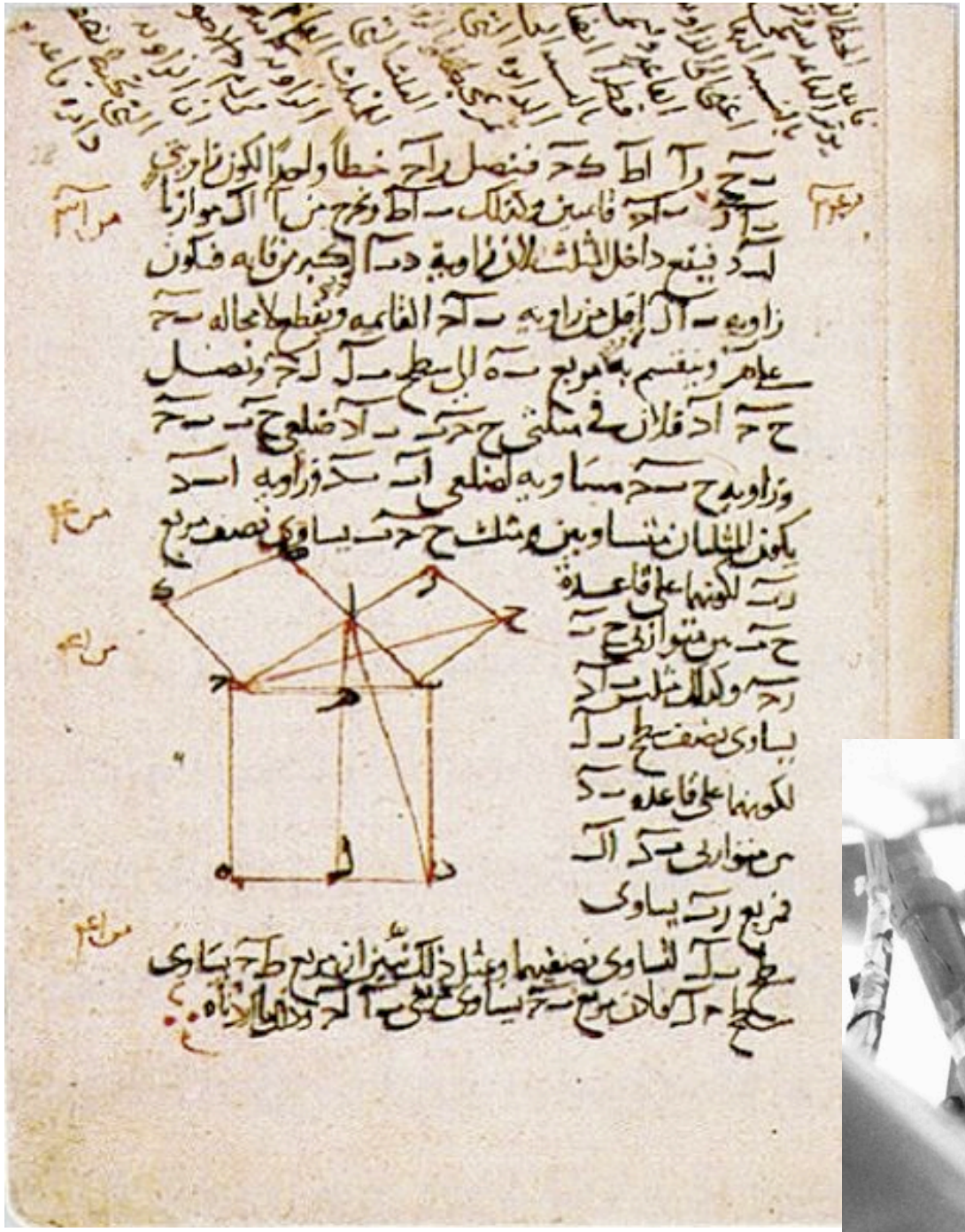


Figure 1 - The figure of proof upon which the original 47th Problem of Euclid is based. This figure is also called "The Bride's Chair". The outline of this figure is used Symbolically in Freemasonry.

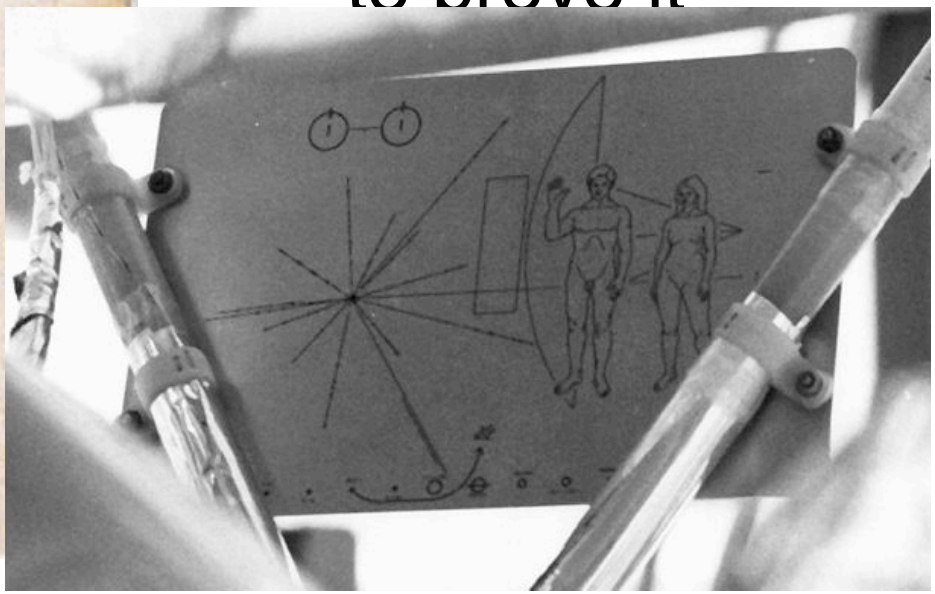
On the throne of Isis nursing Horus we found depicted the classical Geometrical Demonstration of Pythagoras' Theorem



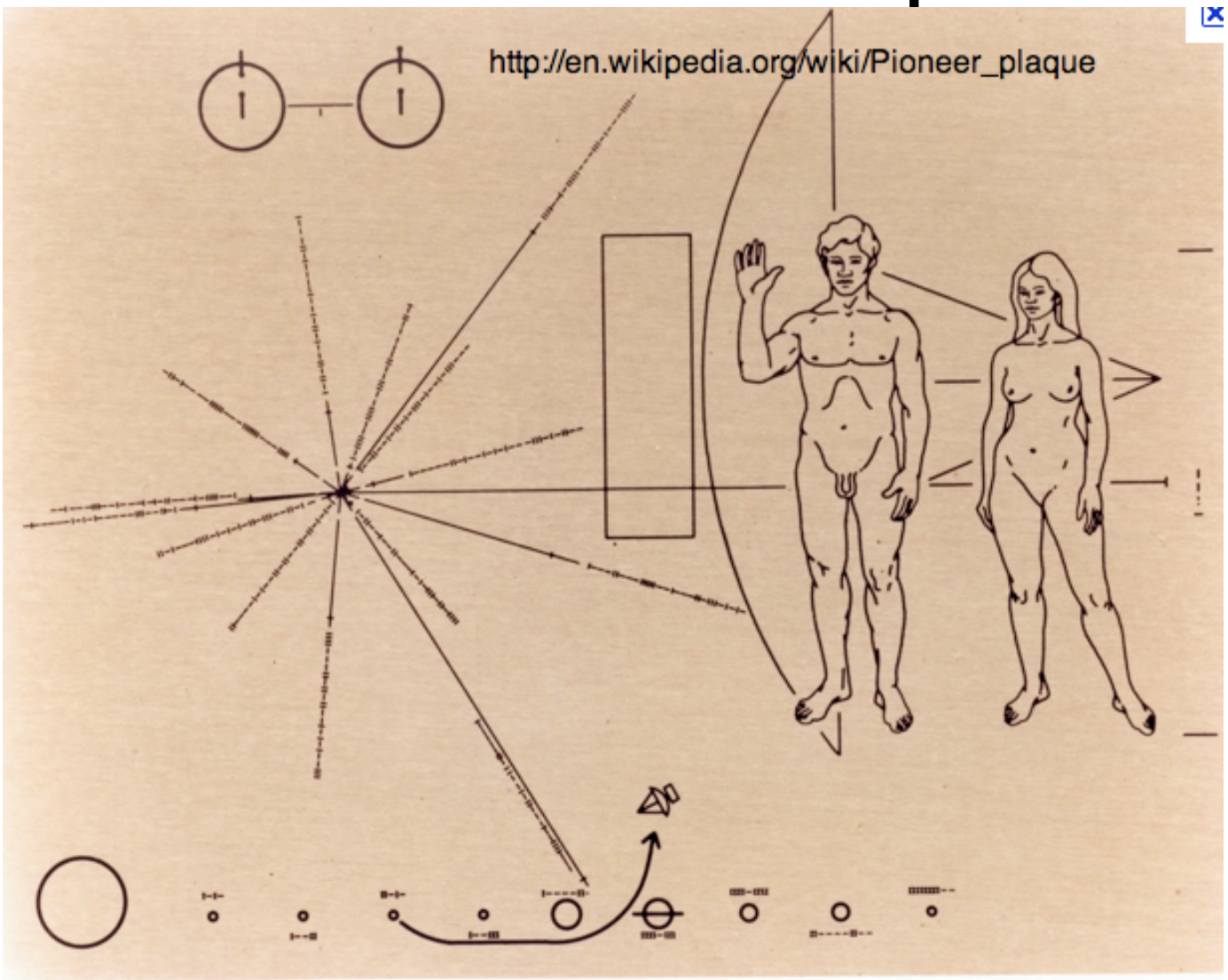
Pythagoras' Theorem according to Google



The picture for Euclid's proof was chosen for the plaque on Pioneer 10 not so much to prove that we knew the theorem, but that we knew how to prove it



Pioneer 10 Plaque

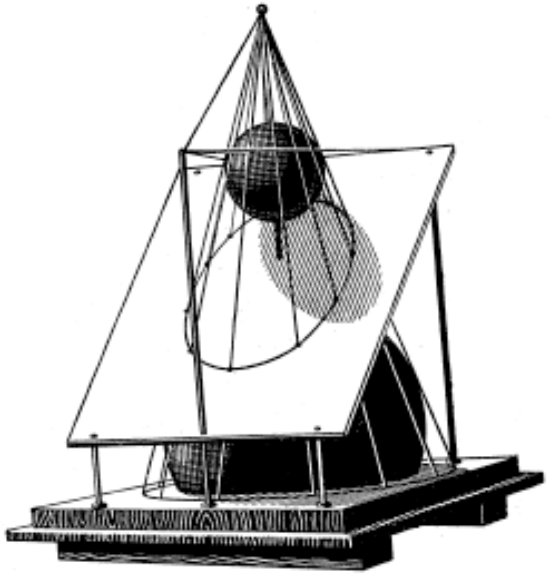


Without the Pythagorean Theorem

VERZEICHNIS

MATHEMATISCHER MODELLE

SAMMLUNGEN H. WIENER UND P. TREUTLEIN



AUS DEM VERLAG VON B. G. TEUBNER

IN LEIPZIG UND BERLIN 1912

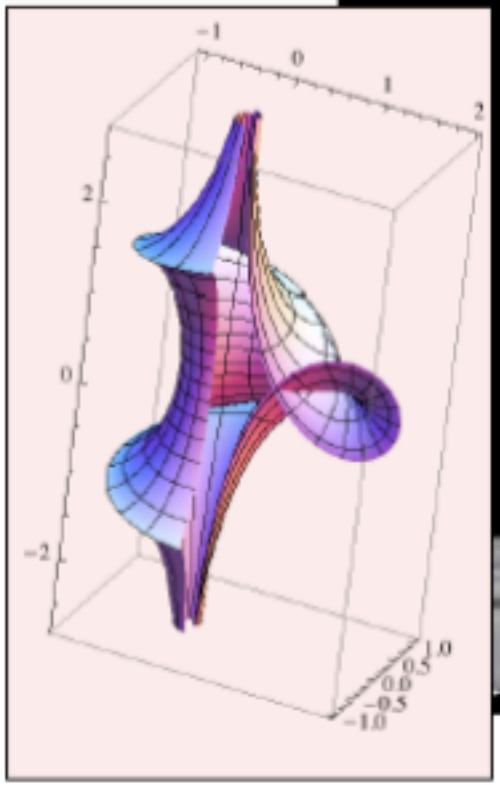
Tori Corkery, Math Calendar 2009

Scanned image

$$x[u\_, v\_] := \frac{2\,(\mathrm{Cos}[u] + u\,\mathrm{Sin}[u])\,\mathrm{Sin}[v]}{1 + u^2\,\mathrm{Sin}[v]^2}$$

$$y[u\_, v\_] := \frac{2\,(\mathrm{Sin}[u] - u\,\mathrm{Cos}[u])\,\mathrm{Sin}[v]}{1 + u^2\,\mathrm{Sin}[v]^2}$$

$$z[u\_, v\_] := \mathrm{Log}\left[\mathrm{Tan}\left[\frac{v}{2}\right]\right] + \frac{2\,\mathrm{Cos}[v]}{1 + u^2\,\mathrm{Sin}[v]^2}$$



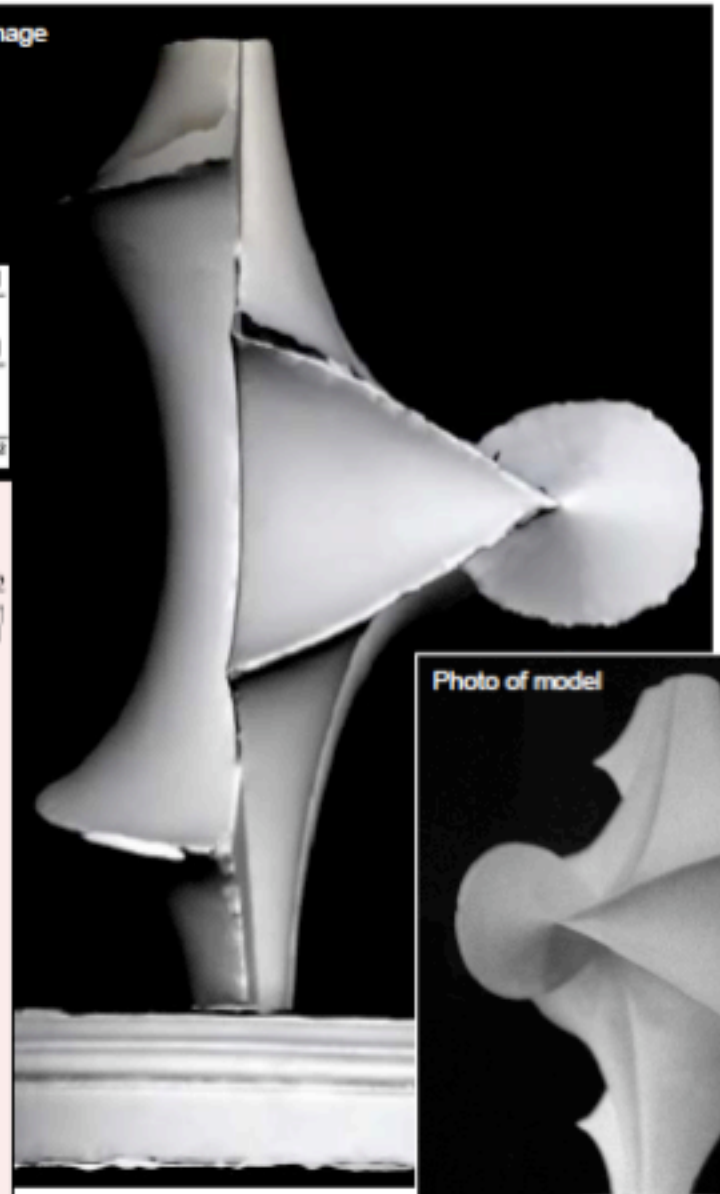
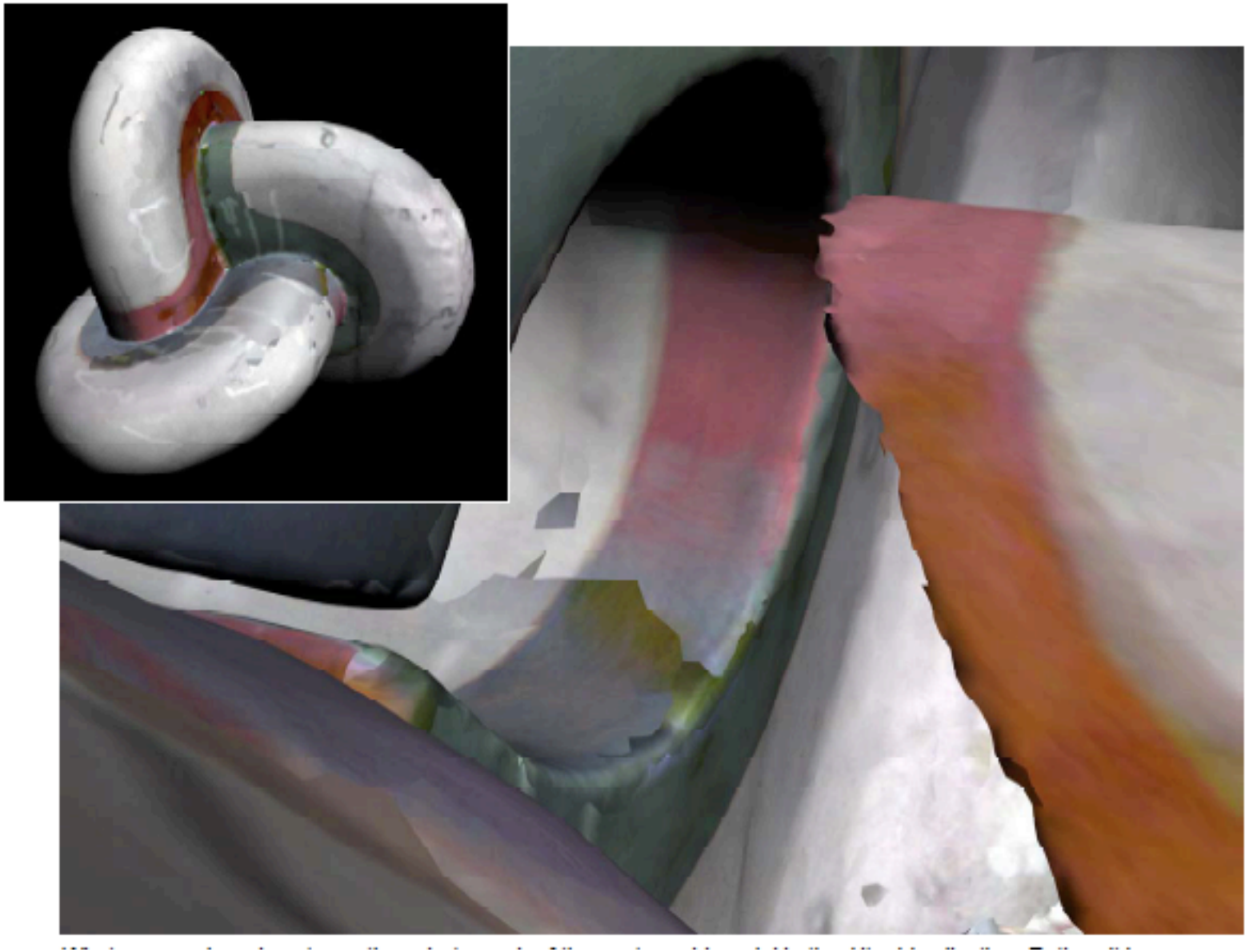


Photo of model

Inside view of Abby Watt's scan of the plaster Boy's surface. Calculart 2008



The Optiverse

SIGGRAPH’98 Electronic Theater Reel
Videomath Reel, ICM, Berlin, 1998.

With
John Sullivan, Stuart Levy,
Ken Brakke, Rob Kusner,
François Apéry, Dana Plepys,
Camille Goudeseune,
Chris Hartman, Alexei Bourd,
Glenn Chappell, Jeff Carpenter.

Collaborators, co-authors, friends

Teachers:

PLATO 1977-1980:

Judy and Bruce Sherwood

Dept of Industrial Design:

Norm MacFarland, Ed Zagorski, Vivian Faulkner-King

Student Assistants:

UIMATH.Applelab:1983-1994

Jim Bailey **Ferrell Wheeler**
Ted Emerson **Cary Sandvig**

REL/CAVE/grafiXlab:1988-2000

Ray Idaszak **Glenn Chappell**
Chris Hartman **Alex Bourd**
Ulises Cervantes-Pimentel
John Estabrook **Matt Hall**

Colleagues:

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NCSA, Urbana 1986-2000:

Donna Cox **Carl Hoyer,**
Bob Patterson **Jeff Carpenter.**

EVL, Chicago 1987-1998:

Dan Sandin **Tom DeFanti**
Maxine Brown **Ellen Sandor**
Dana Plepys **Dave Pape**
Carolina Cruz-Neira

Geometry Center 1989-1997:

Pat Hanrahan, Charlie Gunn, Stuart Levy, John Sullivan

Virtual Environments:

CAVE, CUBE, CANVAS (ISL) **Ben Schaeffer, Jim Crowell, Camille Goudeseune, Hank Kaczmarski**

DiVE (Duke) **Rachael Brady, David Zielinsky**

Portal (TUB) **Samy Khedem, John Sullivan, Steffen Weissman**

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Research Experiences for Undergraduates

Audible Sketchpad (NCSA 1998-2000): **Ande Croll , Jessica Jackson, Doug Nachand, Bob Pinta, Ben Shanbaum, Paul Whitaker, Matt Woodruff.**
illiMath2001 (VIGRE): **Ben Bernard, Ben Farmer, Mark Flider, Doug Nachand, Alison Ortony, Lorna Salaman, Ben Shanbaum, Robert Shuttleworth, Matt Woodruff.**
illiMath2002(VIGRE): **Amit Chatwani, Ben Farmer, Abdul Hamide, Brad Henry, Wendy Hubbard, Yana Malysheva.**
PyCube2004 (Math Dept): **William Baker, Blair Flicker, Emily Gunawan, Greg Stanton, Brett Witt.**
illiMath2006 (REUsite): **Dave Bergman, Nicholas Duchnowski, Emily Echevarria, Matt Gregory, Paul Prue, Chris Rainey, Mimi Tsuruga, Abby Watt.**
illiMath2008 (REUsite): **Chase Boren, Will Davis, Abdul Dakkak, Geoff Ehrman, Lisa Hickock, Sam Ostling, John Pacey, Katie Poon, Liz Rogers.**
illiMath2010 (REUsite): **Chris Bisom, Ian Markwood, Dan Rajchwald, Justin Schirle**
Associated Mentors: **Robert Acar (Puerto Rico), Peter Brinkmann (CCNY), Ulises Cervantes-Pimentel (WRI), Elizabeth Denne (Harvard), Abdul Dakkak (WRI), Paul McCreary (Evergreen), Mike Pelsmajer (IIT), Karen Shuman (Grinnell), Rose Marshack (ISU), Tony Robbin (NY), Jeff Weeks (NY).**

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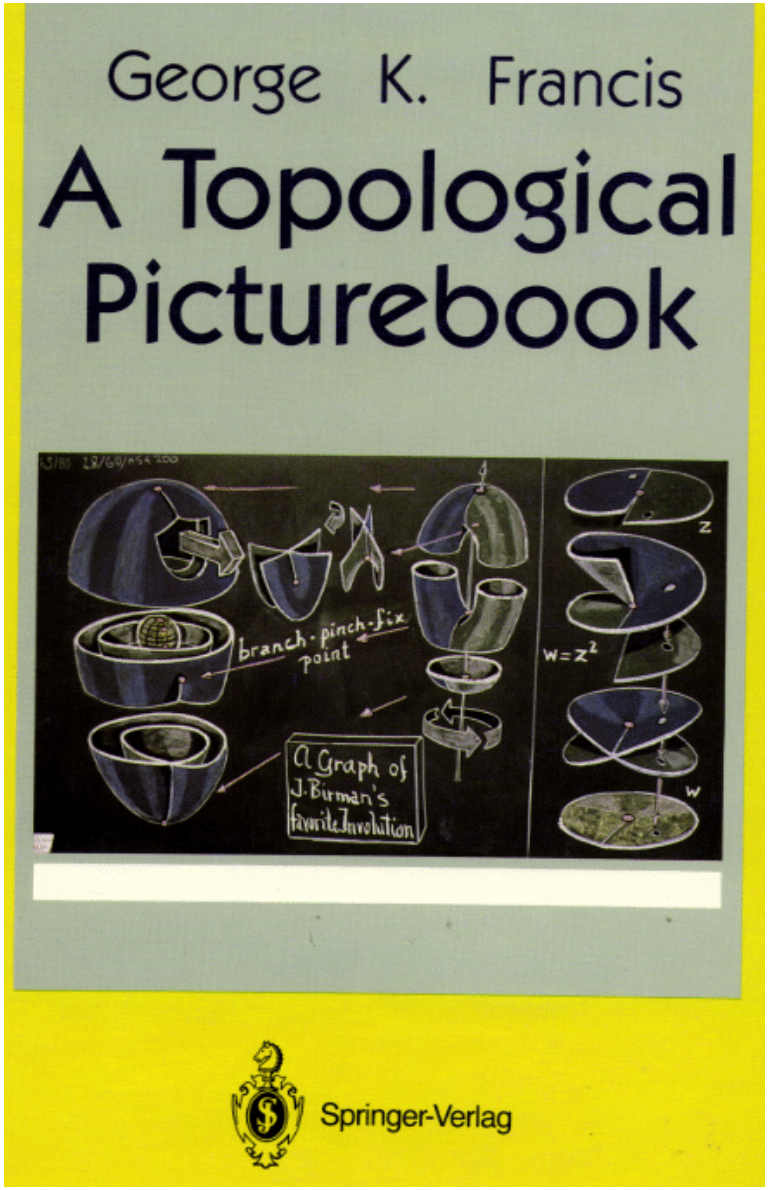
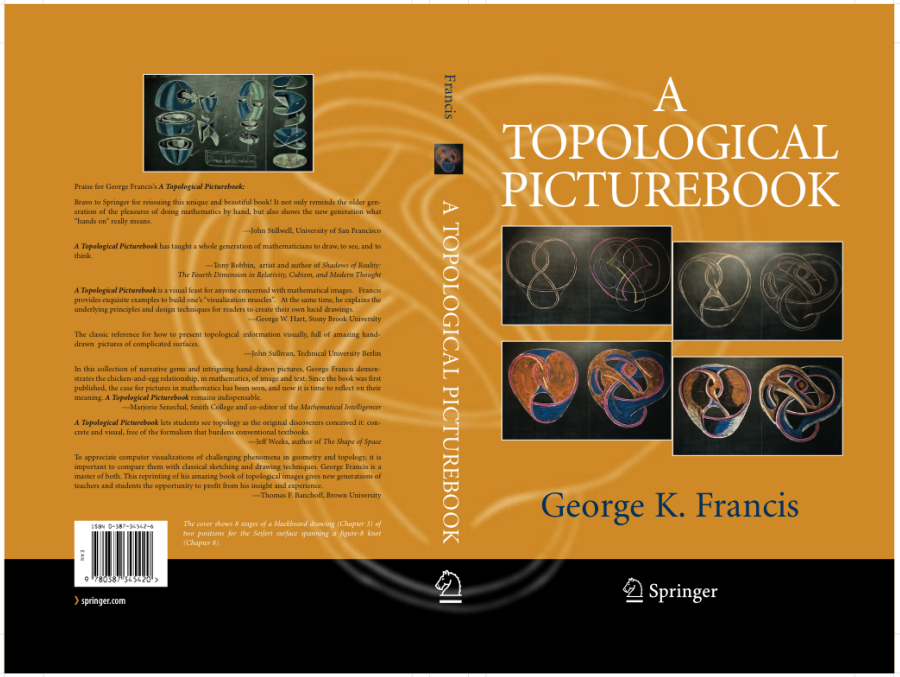

George K. Francis
A Topological Picturebook
branch-pinch-fix point
W=Z²
A Graph of J. Birman's favorite Involution
Springer-Verlag


A TOPOLOGICAL PICTUREBOOK
George K. Francis
Springer
Francis
A TOPOLOGICAL PICTUREBOOK
springer.com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9784431711841
1923041027005
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8
トポロジーの絵本
G.K.フランシス 著
笠原晧司 監訳
宮崎興二 訳
SPRINGER READINGS IN MATHEMATICS
シュプリンガー数学リーディングス 第8巻
A Topological Picturebook
トポロジーの絵本
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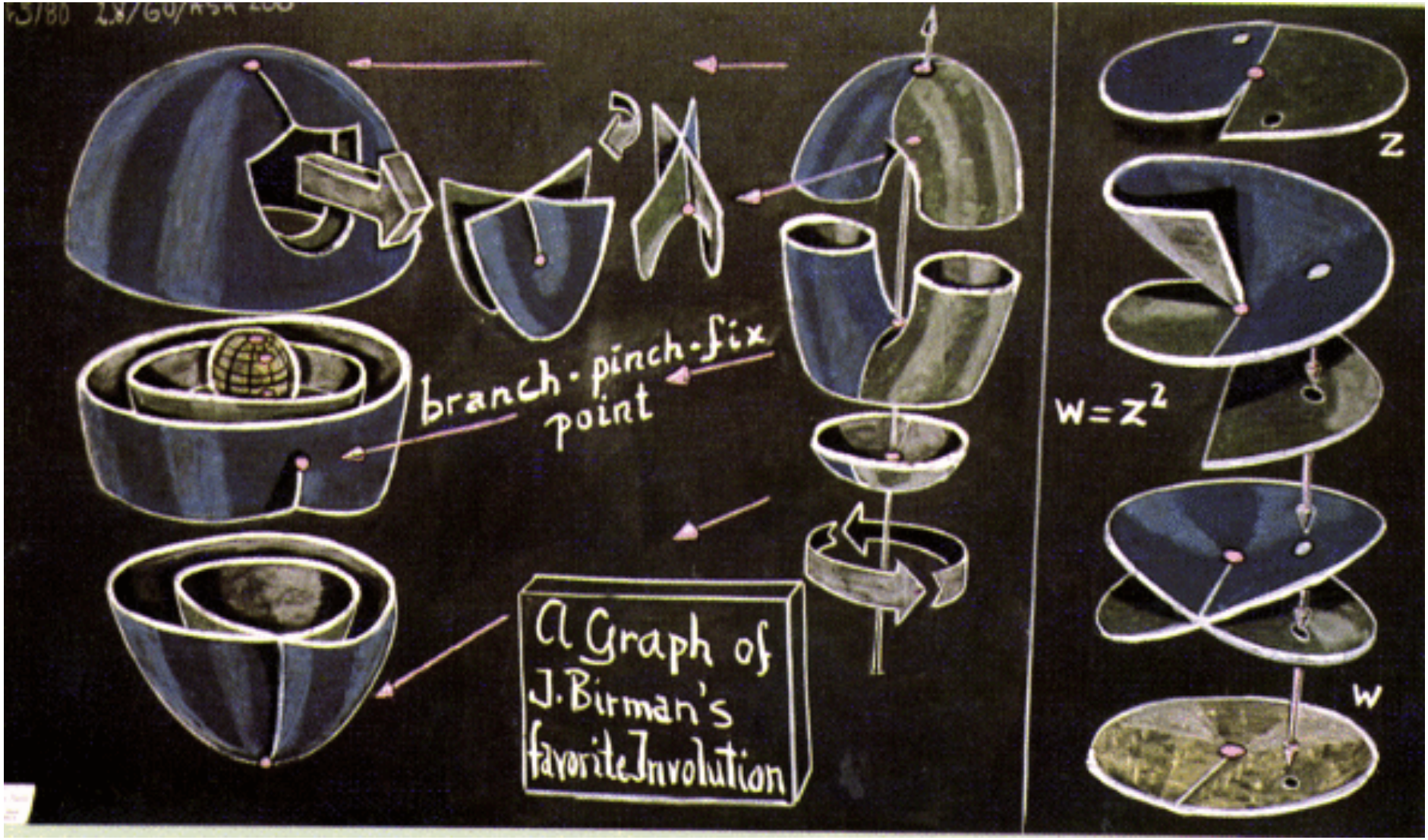
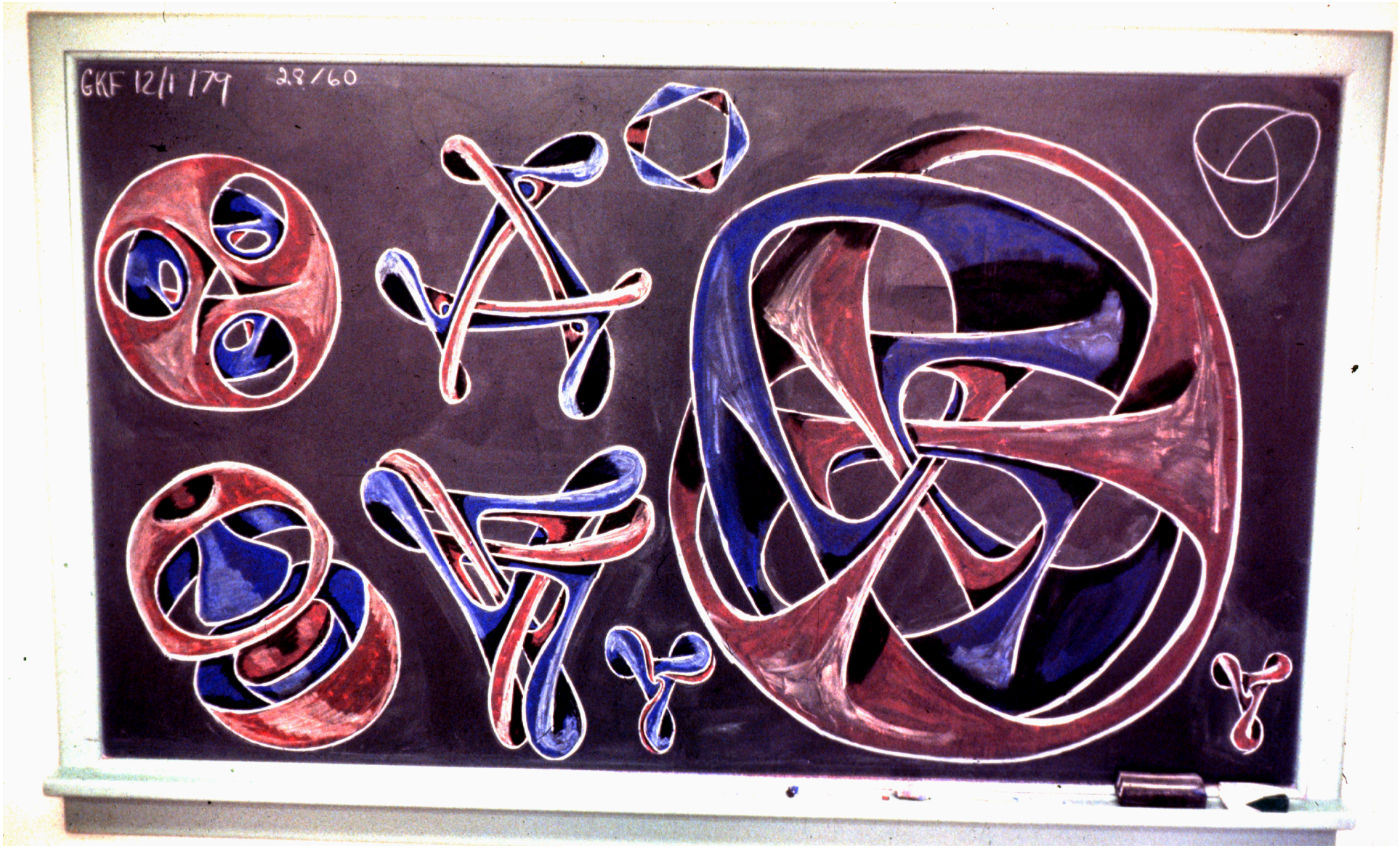
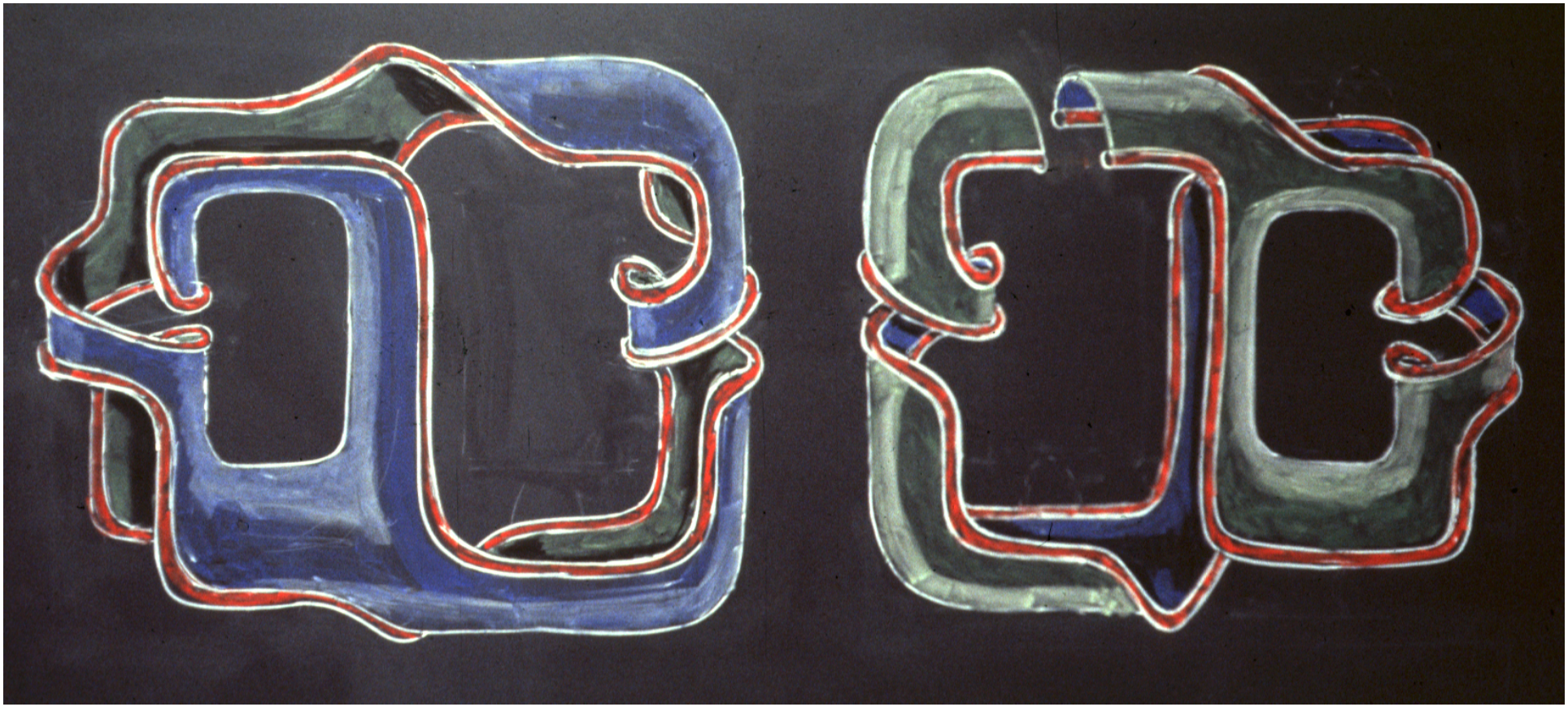
Joan Birman's Favorite Involution





Unsymmetrical Triblague 1979



Isotopic surfaces spanning a Figure-8 knot

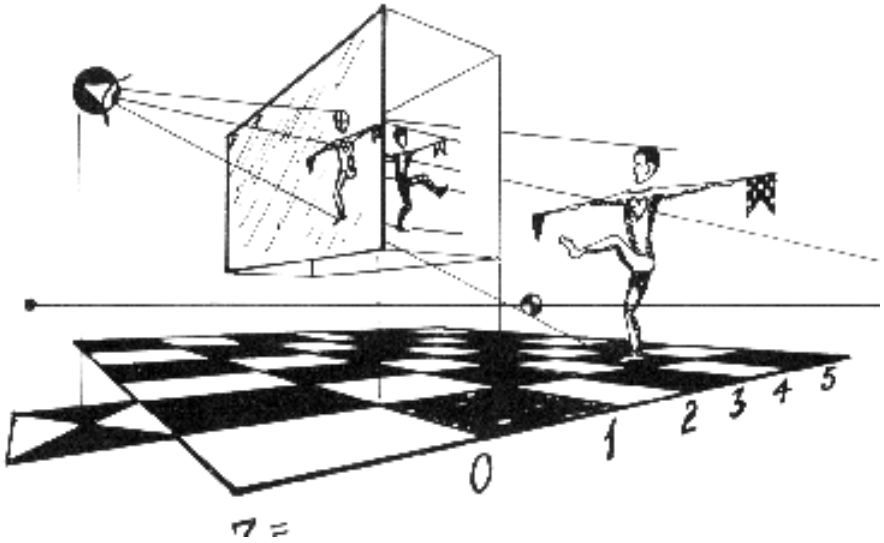
Some Content Now, Please

This is a math course.

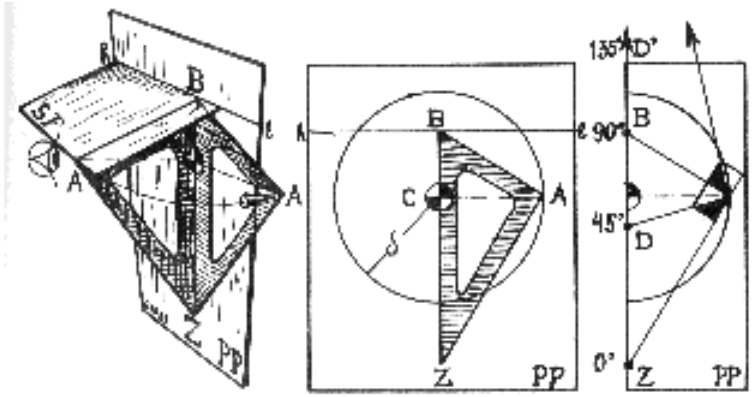
Already!

Linear Perspective

Renaissance
and
OpenGL



Horizon – Zenith
and the
Geometric Mean

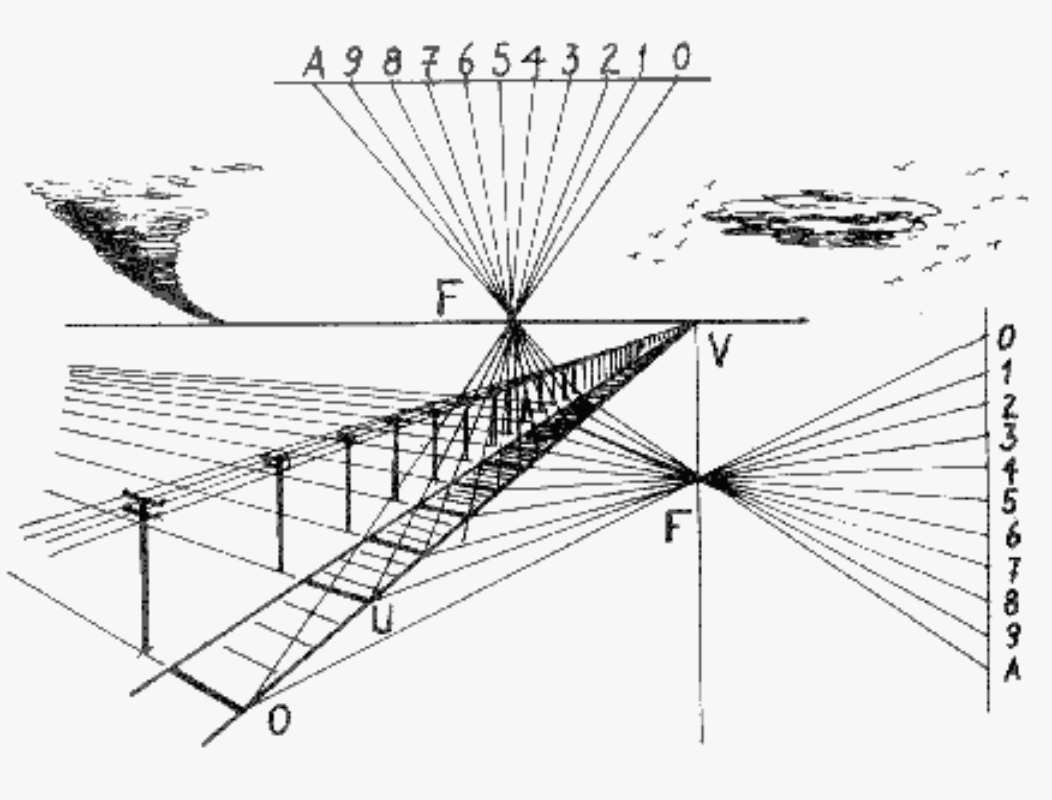


from George K. Francis, “A Topological Picturebook”, Springer-Verlag, 1987

Albrecht Dürer “Reclining Nude”

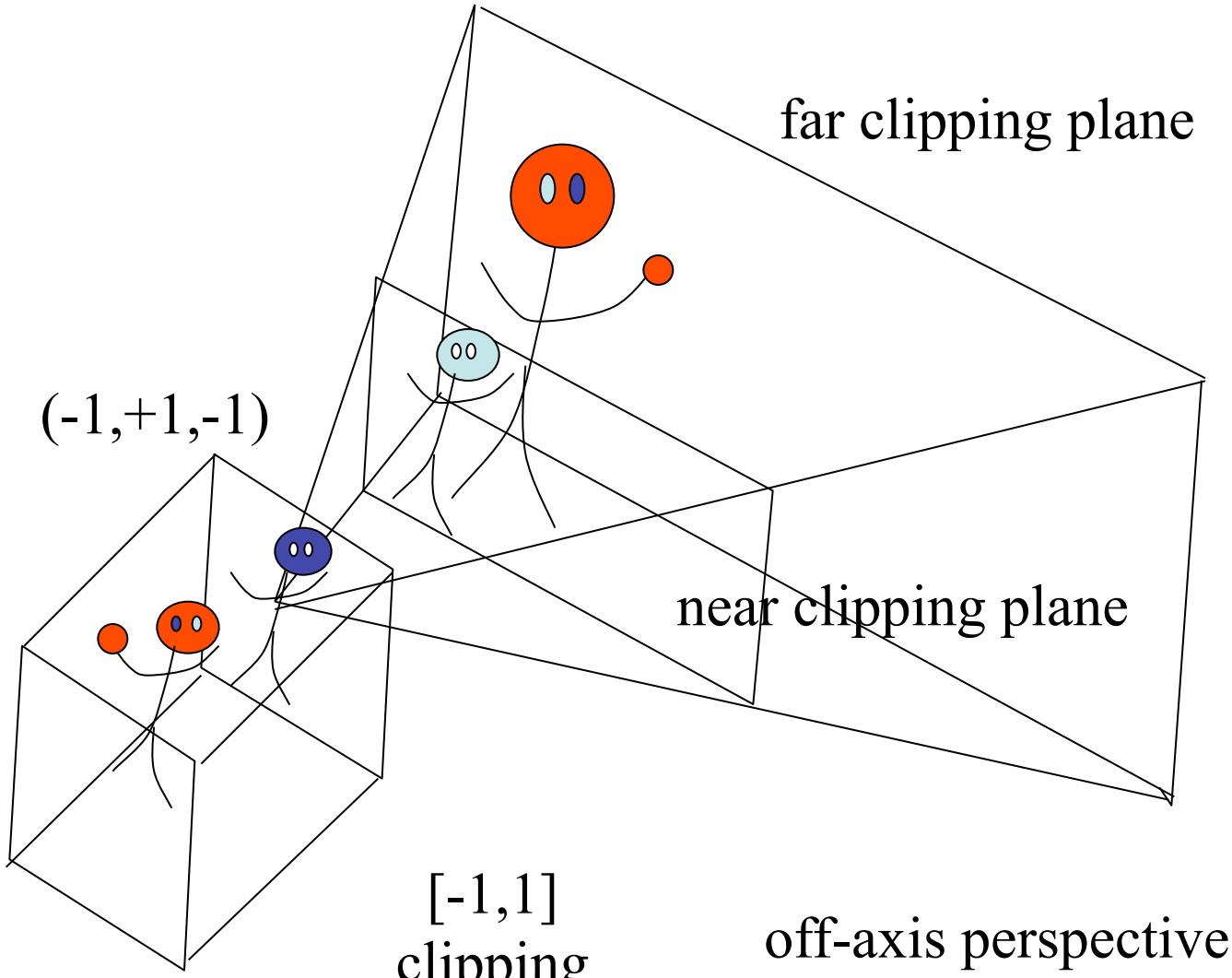
Contemporary version of Alberti's Veil

Prairie Horizon

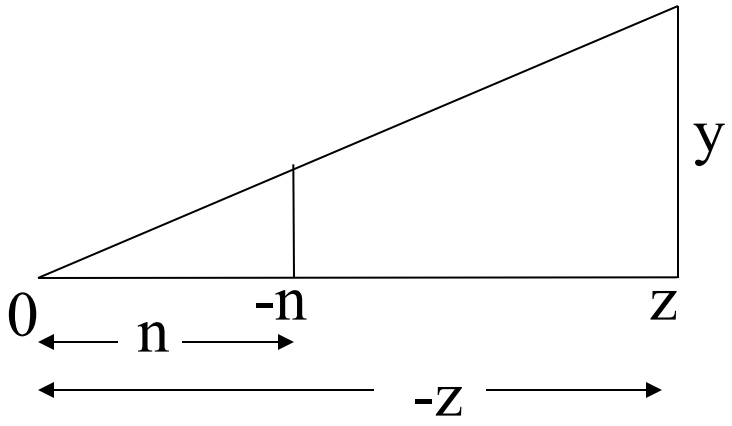


Crossratios are the rulers in projective geometry

OpenGL Frustum



$$\begin{bmatrix} \frac{2n}{r_0-\ell_0} & 0 & \frac{r+\ell}{r-\ell} & 0 \\ 0 & \frac{2n}{t_0-b_0} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & -\frac{f+n}{f-n} & -\frac{2fn}{f-n} \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} \frac{2n}{r_0-\ell_0}x + \frac{r+\ell}{r-\ell}z \\ \frac{2n}{t_0-b_0}y + \frac{t+b}{t-b}z \\ -\frac{f+n}{f-n}z - \frac{2fn}{f-n}w \\ -z \end{bmatrix} \begin{bmatrix} \frac{\left(\frac{nx}{-z}\right)-c_x}{s_x} \\ \frac{\left(\frac{ny}{-z}\right)-c_y}{s_y} \\ -\frac{f+n}{f-n} + \frac{2f}{f-n}\left(\frac{nw}{-z}\right) \end{bmatrix}$$



subject to the OpenGL clipping convention

$$z < -\frac{f+n}{f-n}z - \frac{2fn}{f-n}w < -z$$

whence $z < 0$ and $0 < n < f \implies n < -z < f$

but $f = -n < 0$ everts the clipping box.

frustum in homogeneous coords

frusta

Euclidean : far = infinite

$$\begin{bmatrix} \frac{2d}{r-\ell} & 0 & \frac{r+\ell}{r-\ell} & 0 \\ 0 & \frac{2d}{t-b} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & -1 & -2n \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Hyperbolic: far = 1

$$\begin{bmatrix} \frac{2n}{r_0-\ell_0} & 0 & \frac{r+\ell}{r-\ell} & 0 \\ 0 & \frac{2n}{t_0-b_0} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & -\frac{1+n}{1-n} & -\frac{2n}{1-n} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Spherical: front hemisphere

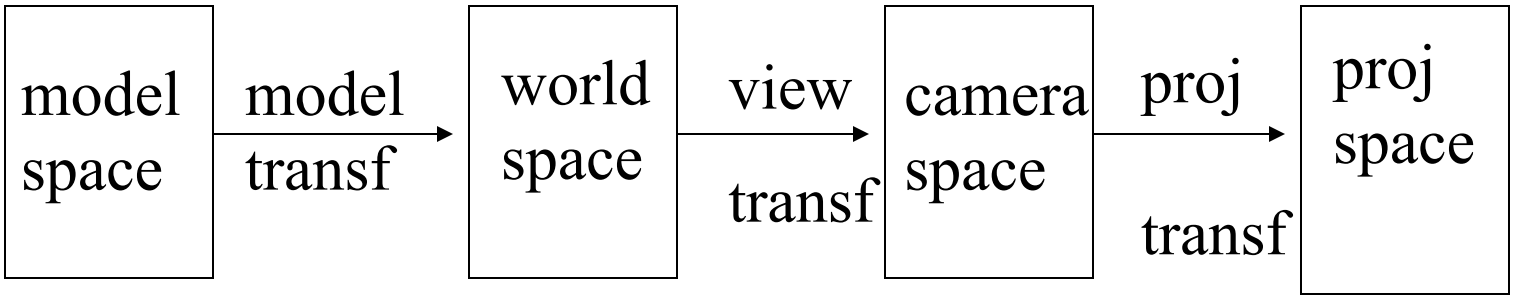
$$\begin{bmatrix} \frac{2n}{r_0-\ell_0} & 0 & \frac{r+\ell}{r-\ell} & 0 \\ 0 & \frac{2n}{t_0-b_0} & \frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{n}{2} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Spherical: back hemisphere

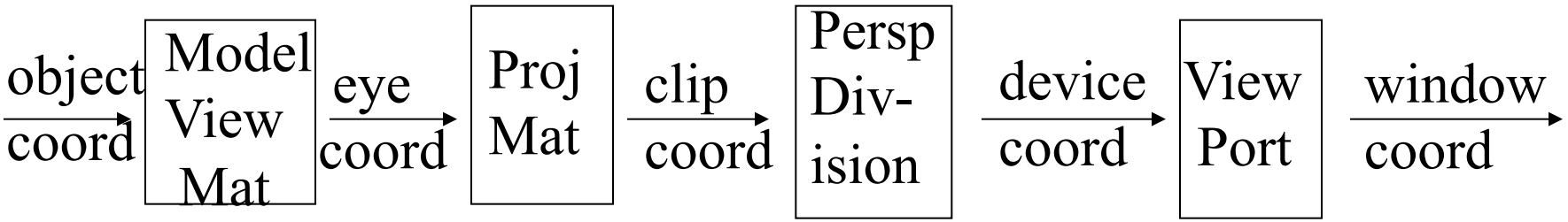
$$\begin{bmatrix} -\frac{2n}{r_0-\ell_0} & 0 & -\frac{r+\ell}{r-\ell} & 0 \\ 0 & -\frac{2n}{t_0-b_0} & -\frac{t+b}{t-b} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{n}{2} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

J. Weeks “Real-Time Rendering in Curved Spaces”, IEEE CGA Nov02

Jeff Weeks, “Real-Time Animations Hyperbolic Spherical, and Product Geometries” in BolyaiFestschrift, to appear soon.



Alias vs. Alibi as in Birkhoff & MacLane



Kurt Akely & Mark Segal, “The OpenGL GraphicSystem”, Samizdat, Dec 1995

Homotopy Escalator

Motion: as in rotations and translations in Euclidean geometry. (Non-Euclidean motions don't belong here for technical reasons.)

Articulation: as in windup toys, puppets, marionettes (The most common sort of animation.)

Distortion: of space itself, carrying everything along with it. (Non-Euclidean motions – isometries – belong here.)

Deformation: every point moves in time and space according some schedule dictated by mathematical formulas, or by a succession of display lists.

"Metarealistic Rendering of Real-time Interactive Computer Animations" in Visual Mind 2

Four Dimensions

4D-viz : We *visualize* phenomena in 4 isotropic dimensions by comparing them to lower dimensional analogues.

Insertions: parametrized curves and surfaces in R^n, n>2.

Mappings: as is mercator projections, waving flags etc.

Shadows: perspective, axonometric, orthographic projections.

Slices: as in floor plans, cross-sections.

4D-fx: We observe the special effects and distortions that 4D phenomena have on 3D scenes.

"Metarealistic Rendering of Real-time Interactive Computer Animations" in Visual Mind 2

Real-Time Interactive Computer Animations

Special Effects:	RTICA
Projections of a 4D motion:	*soniCube*
3D slices of a 4D structure:	*Maze4D*
The 4D dialectic:	*Illusion*
4D-mediated homotopies:	
Conformal map $S^3 \rightarrow R^3$:	*illiSnail*
Navigating curved space:	*illiSpace*
Special Relativity:	*schpRel*
Quaternions in action:	*illiTangle*
Air on the Strings of Dirac (video)	

"Metarealistic Rendering of Real-time Interactive Computer Animations" in Visual Mind 2

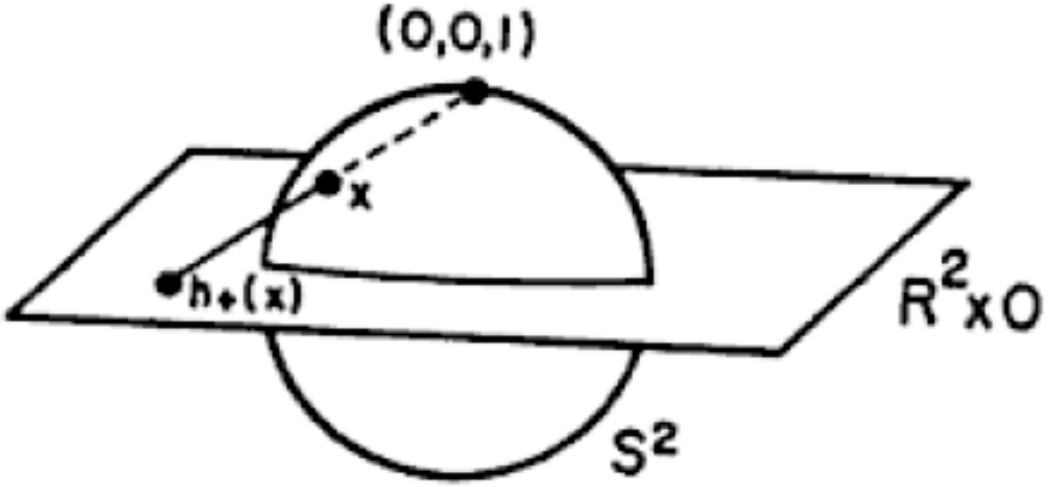
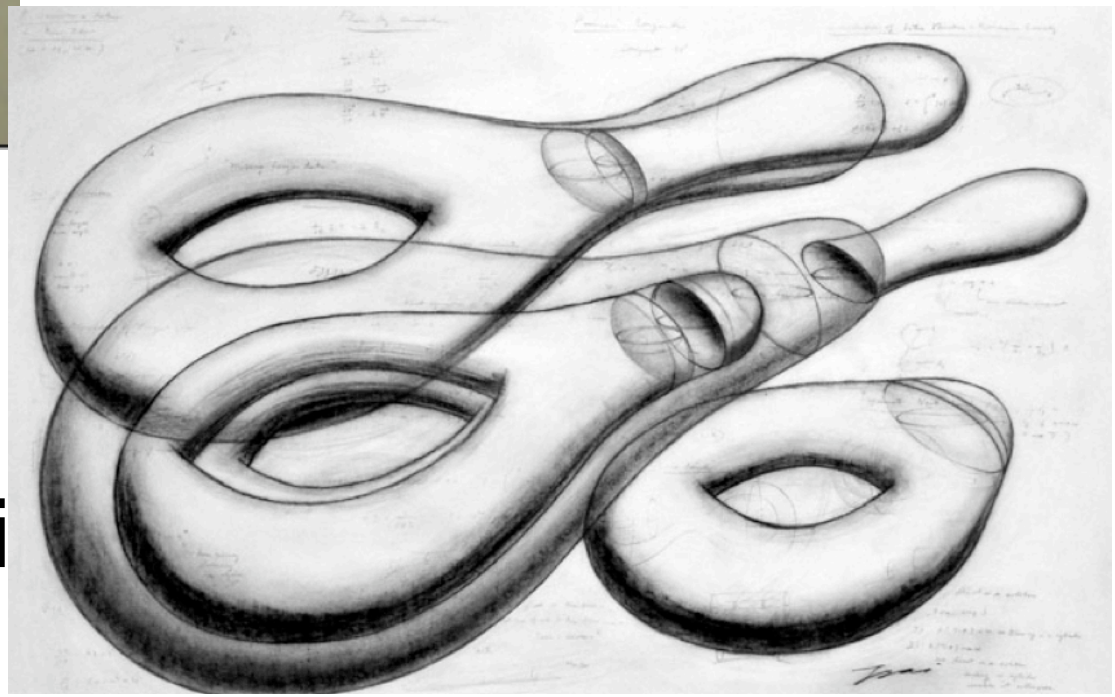


Figure 3. Stereographic projection

Milnor, Topology from the Differentiable Viewpoint, Princeton P

Lun Yi Tsai



Whitney Umbrella

original right
and
elaborated by
Lun Yi Tsai
below

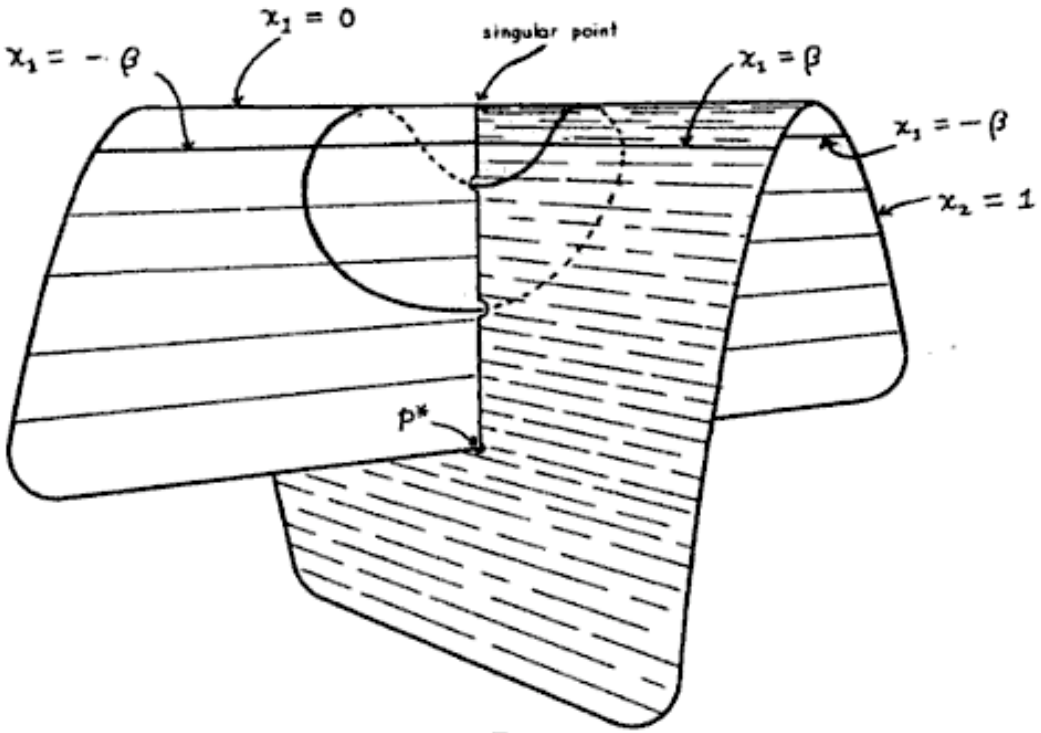


FIG. 1

4. Whitney Family
2008, charcoal and graphite on paper, 39 x 39 in

The Whitney Umbrella, a topological surface named for the American mathematician Hassler Whitney, can be thought of as a plane that is cut along a ray and glued back in such a way that it intersects with itself in three dimensions. This parameterization creates a stack of umbrellas sitting on their sides.

ANNALS OF MATHEMATICS
Vol. 45, No. 2, April, 1944

THE SINGULARITIES OF A SMOOTH n-MANIFOLD IN $(2n-1)$-SPACE*

BY HASSLER WHITNEY

(Received August 19, 1943)

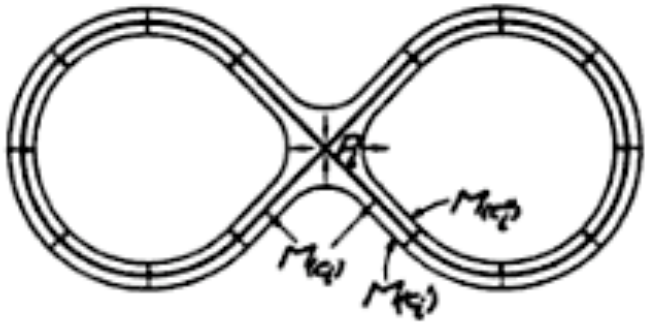


FIG. 1. Point p_i of type $1 \to 2$. Small arrows show direction of increase of f_4

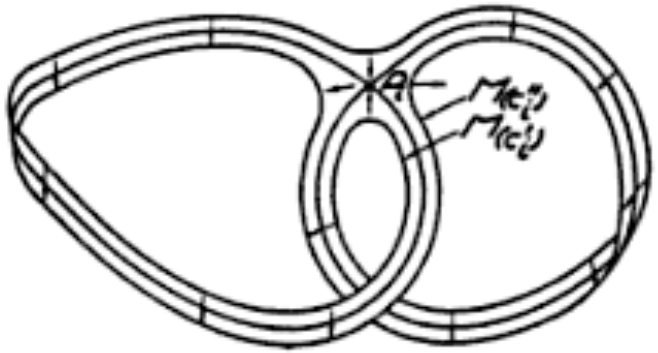


FIG. 2. Point p_i of type $1 \to 1$

Morse Theory

Boy's Surface (immersed projective plane) by sections

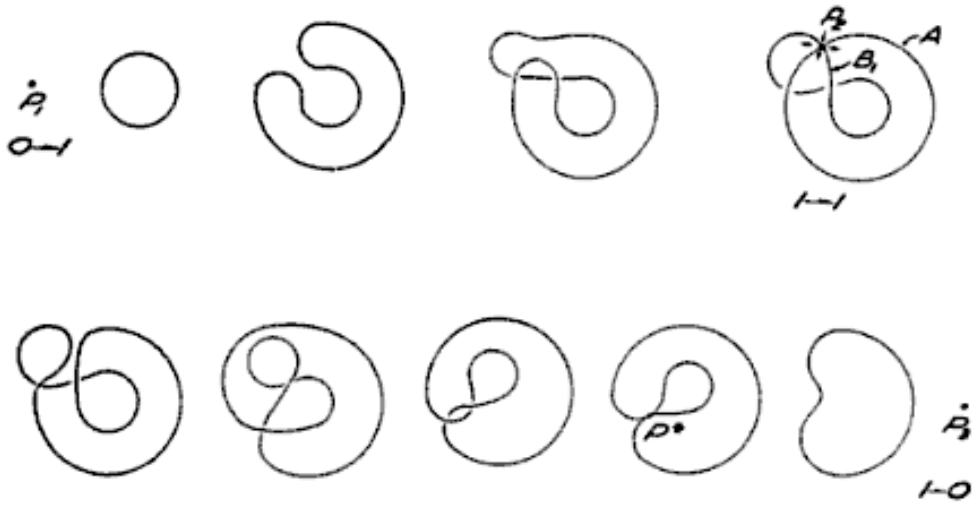


FIG. 6

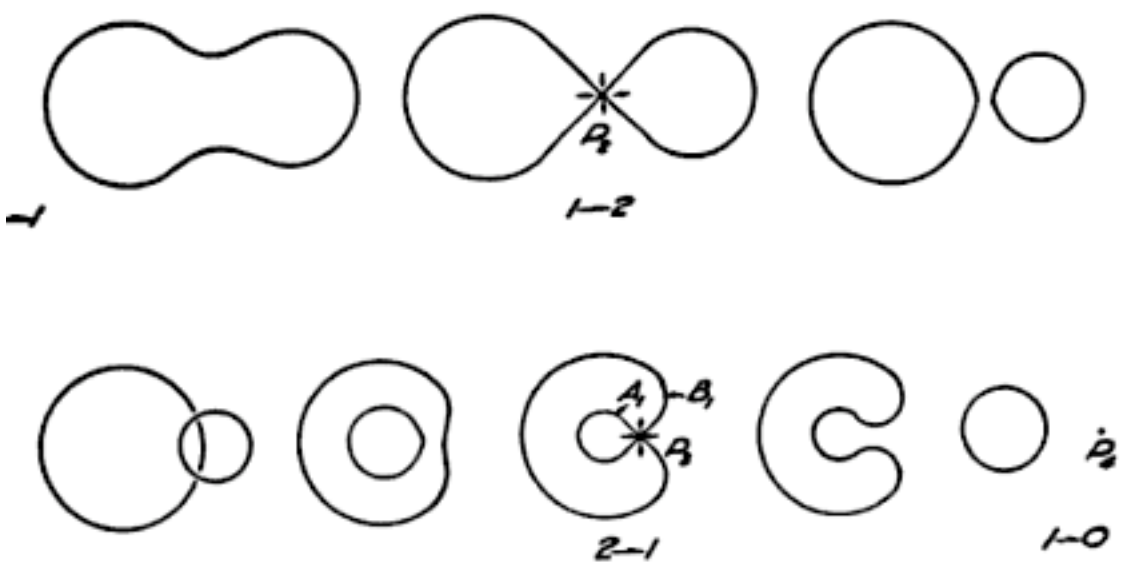


FIG. 3

Kleinbottle sections by Whitney

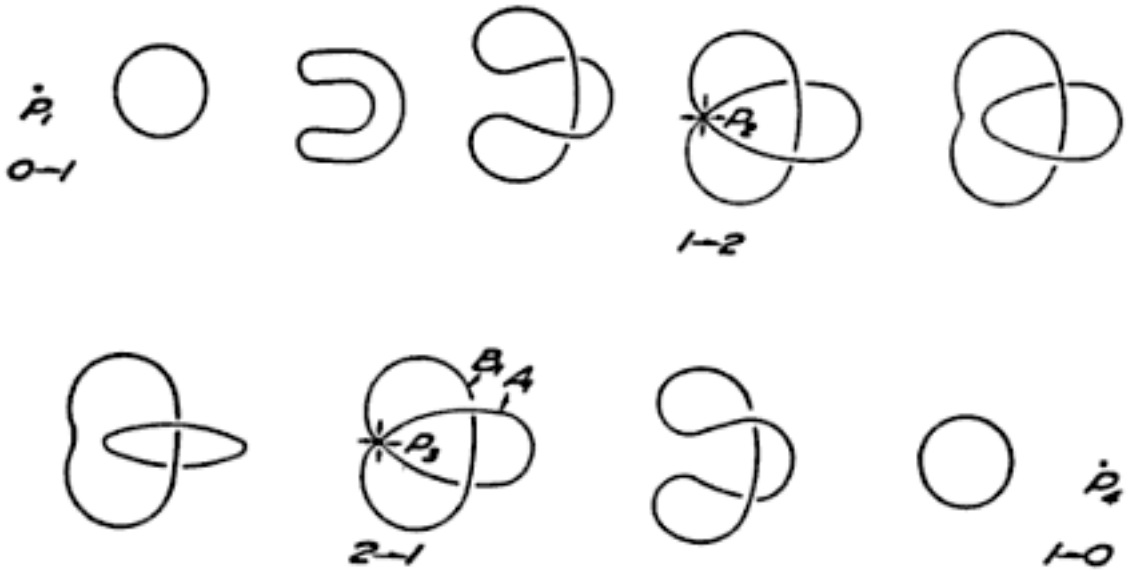
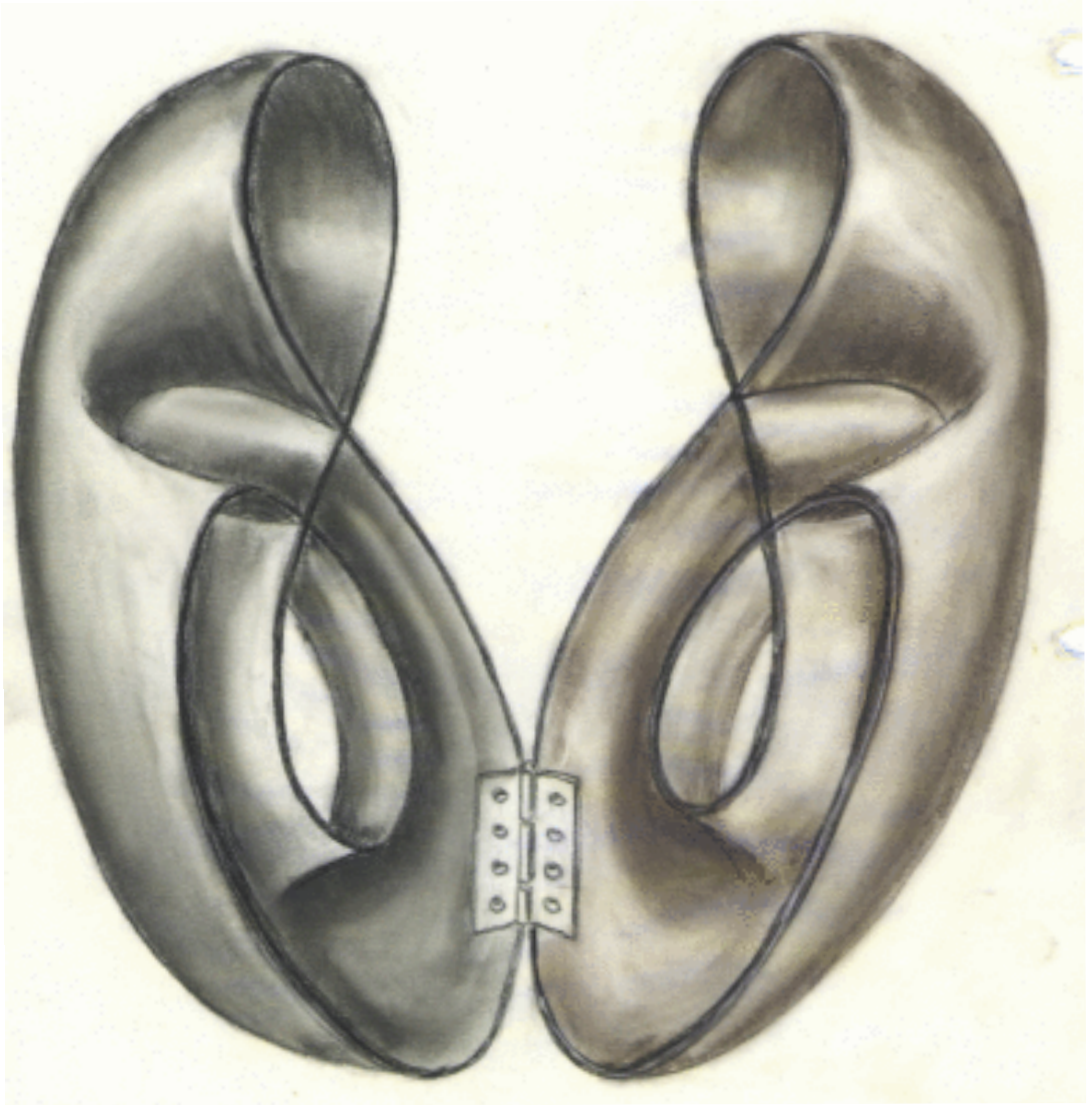


FIG. 4

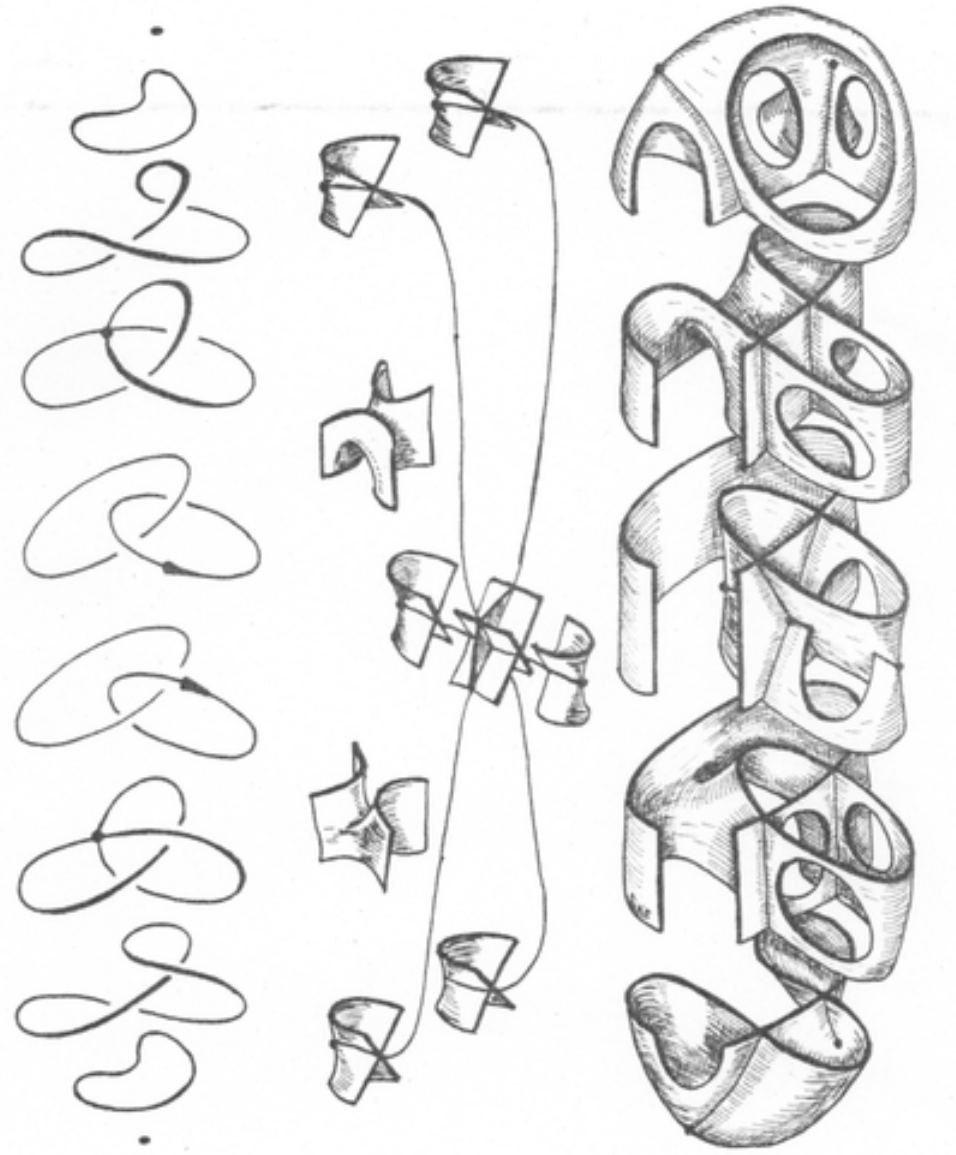
Hinged Kleinbottle



Pastel drawing of two immersed Möbius bands that form a Kleinbottle, 1983.

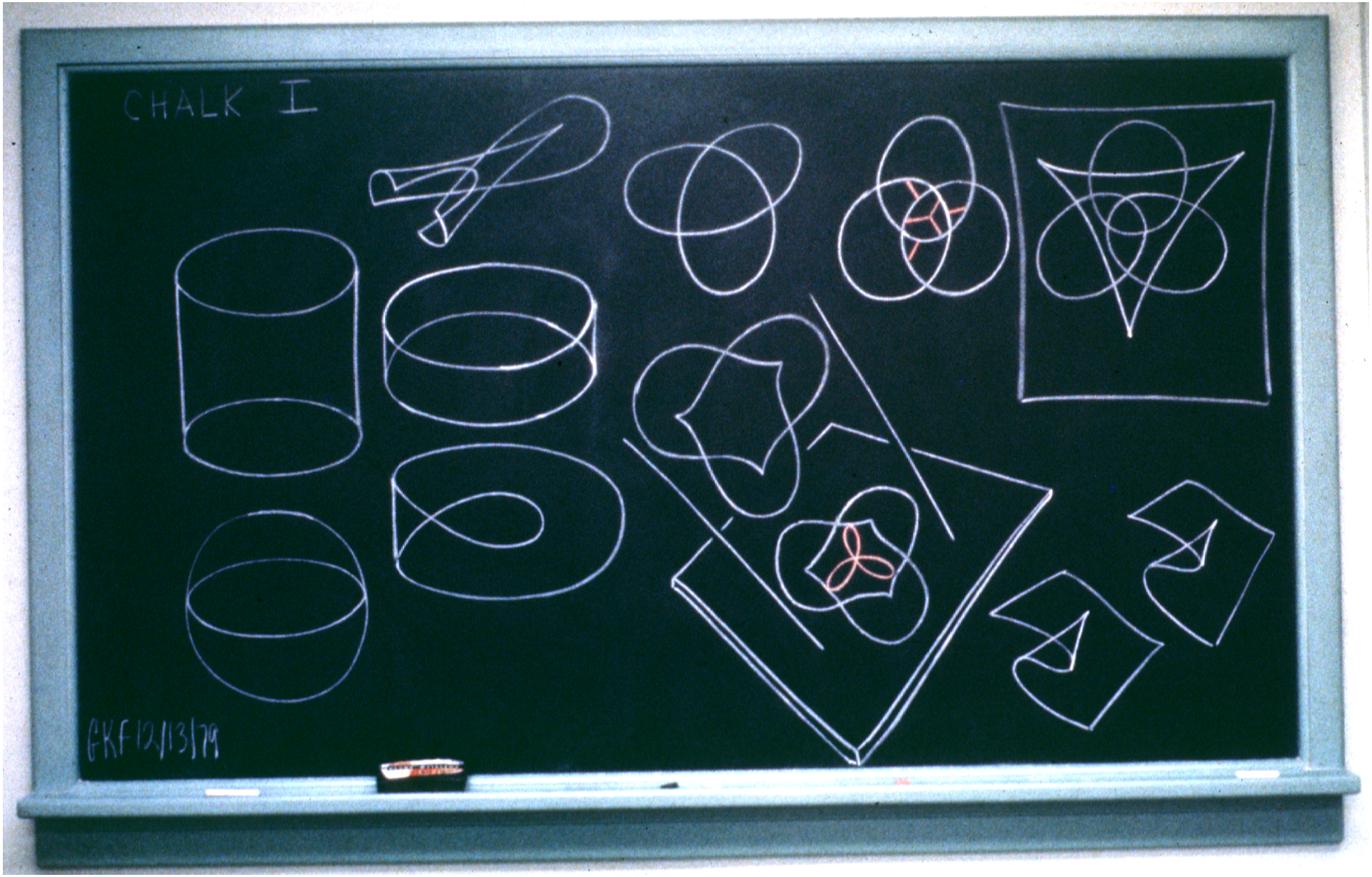
Whitney Bottle

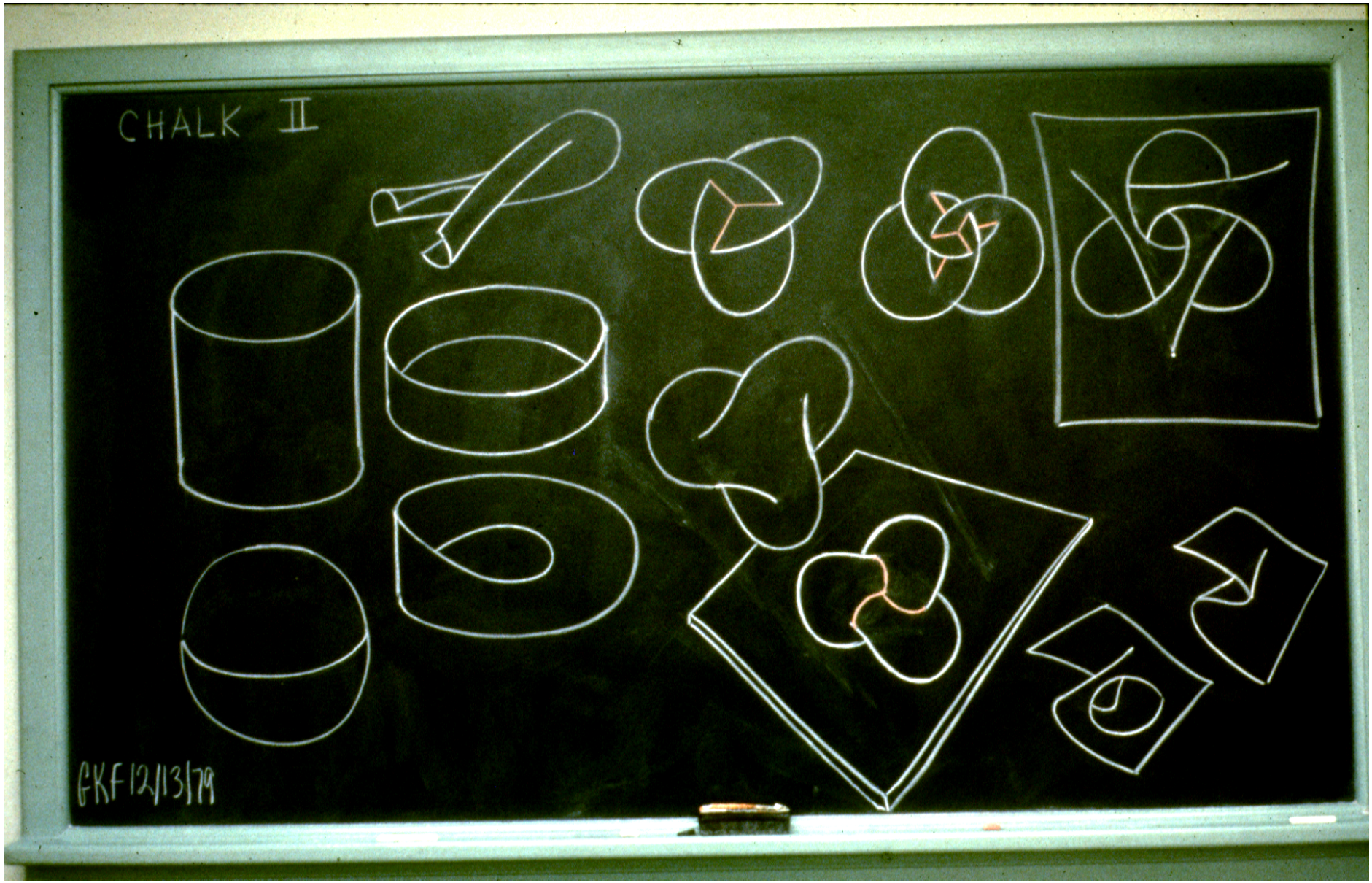
From
3-D sections
construct the
double locus,
& shadow of
a Kleinbottle
embedded in
4-Space

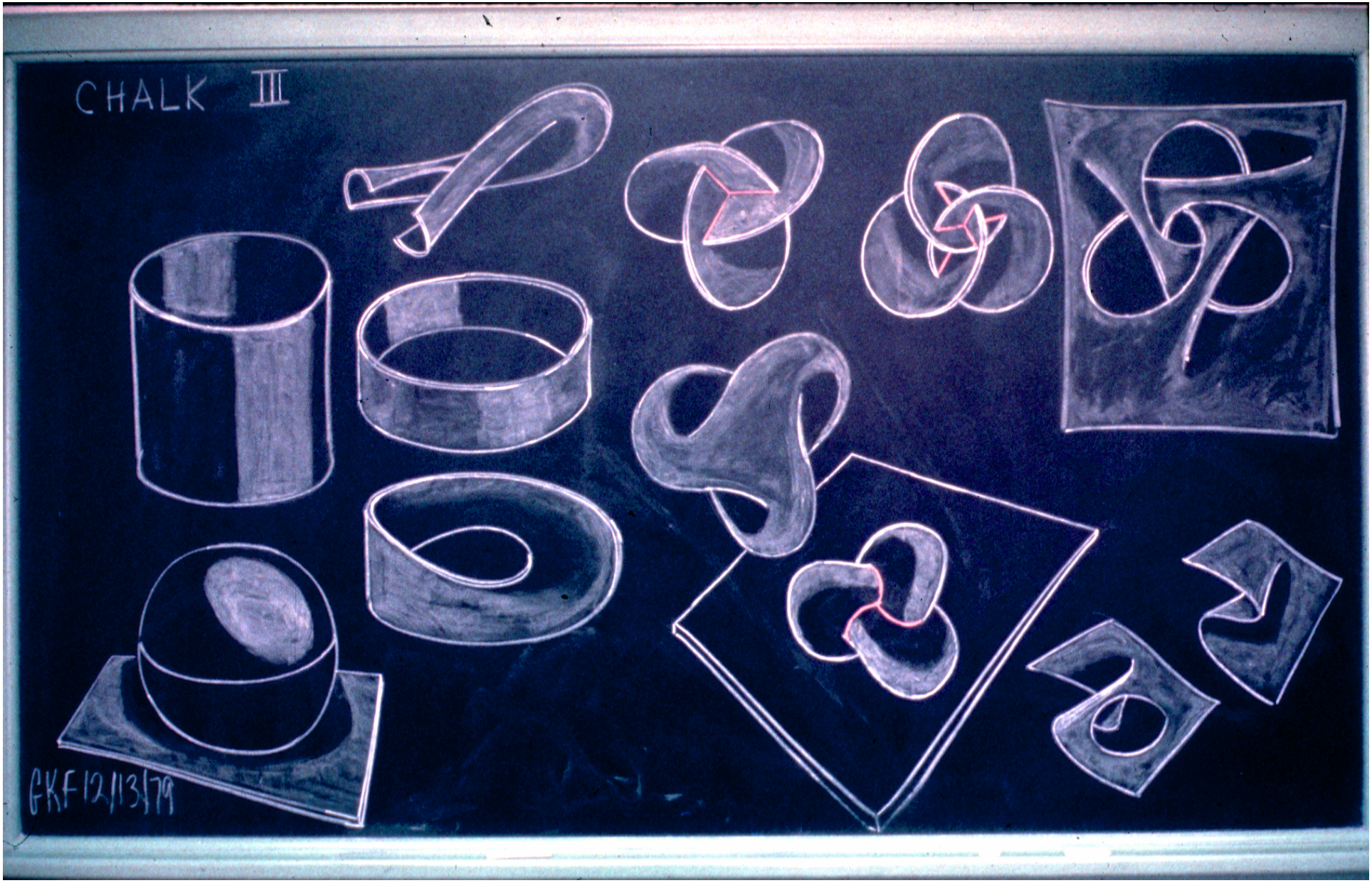


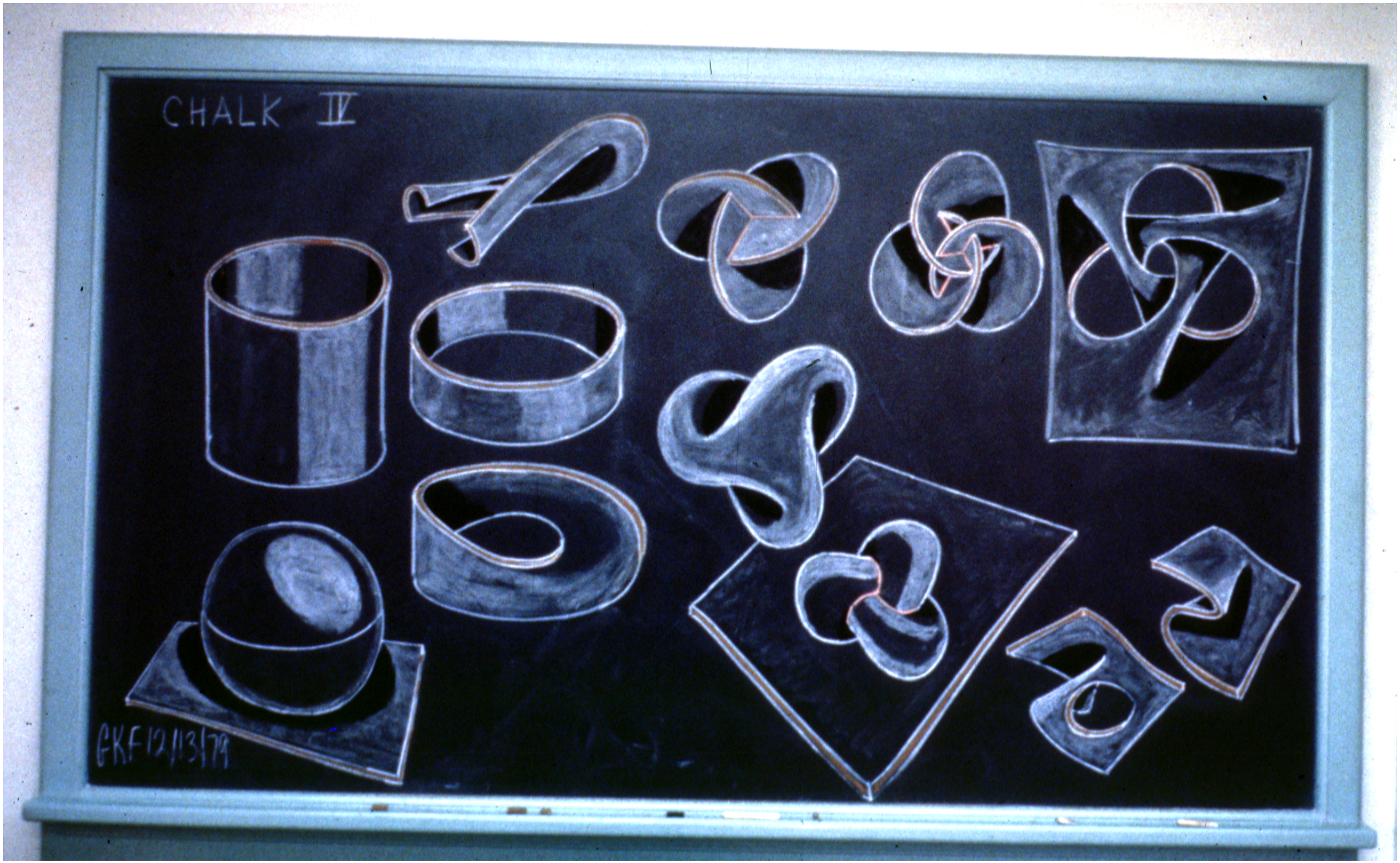
sections double shadow

from George K. Francis, “A Topological Picturebook”, Springer-Verlag, 1987


CHALK I
FKF 12/13/79


CHALK II
GKF 12/13/79


CHALK III
GKF 12/13/79


CHALK IV
GKF 12/13/79

Blackboard Dunce Cap

A blackboard drawing goes through 4 stages:

Draw all lines to make a template.

Hide lines for a line drawing (& windows).

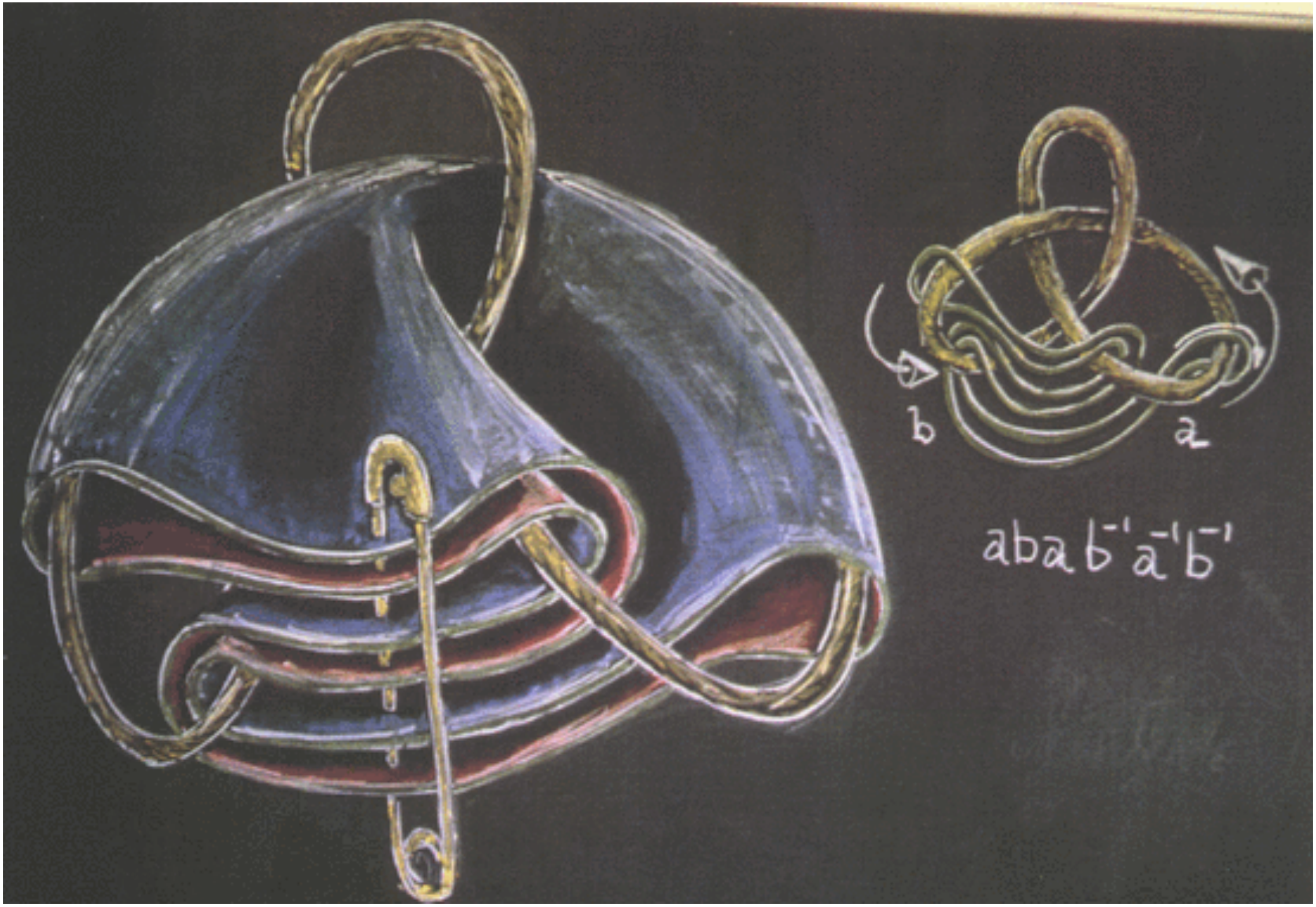
Light and shade for monochrome picture.

Coloring and labeling for information.

Make a photograph before erasing!

from George K. Francis, “A Topological Picturebook”, Springer-Verlag, 1987

Diapered Trefoil Knot



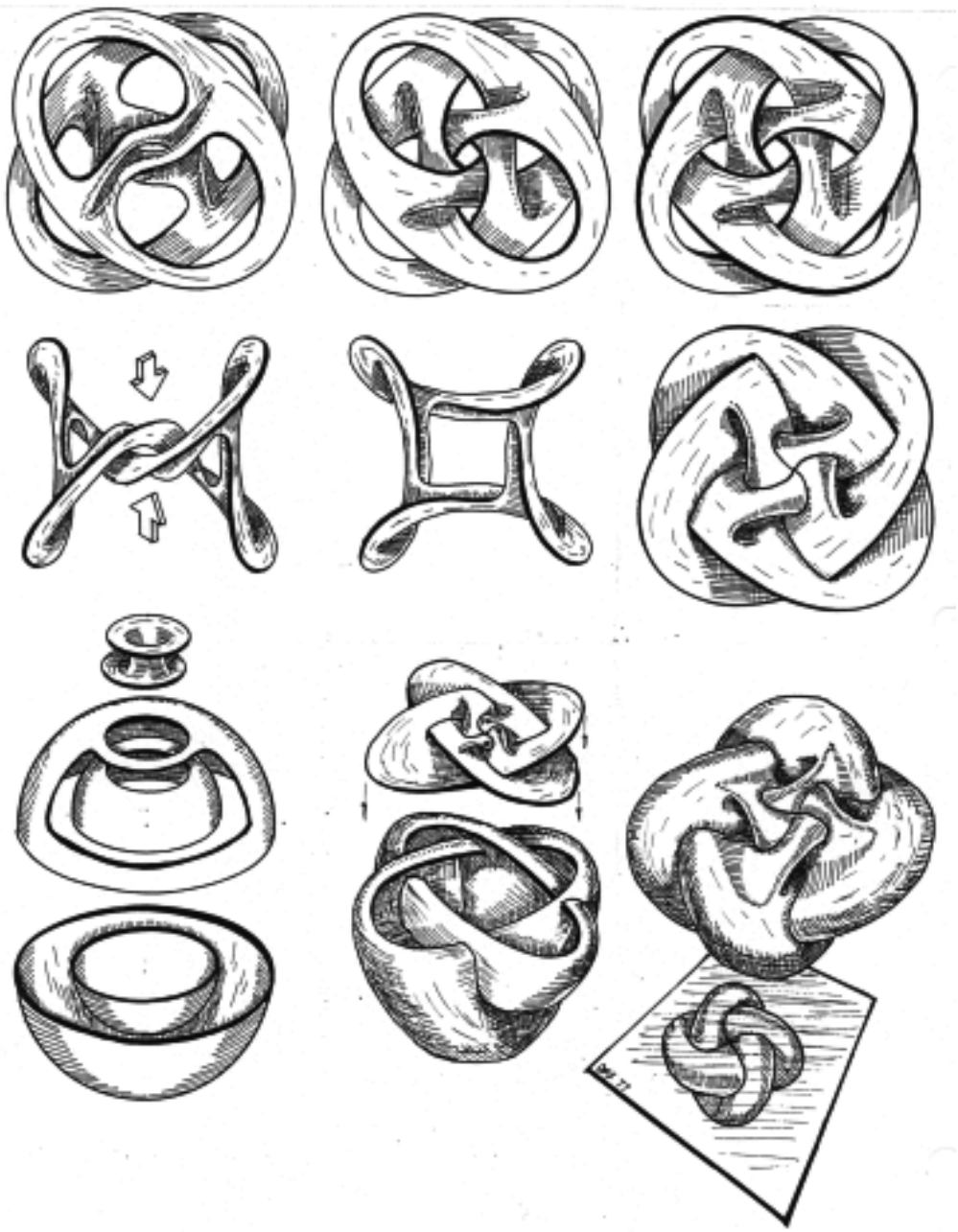
from George K. Francis, “A Topological Picturebook”, Springer-Verlag, 1987

Five Epochs of Sphere Eversions

Prehistory: Werner Boy (1901) anticipates the
Whitney-Graustein Theorem (1937) which is
generalized by S. Smale (1957) who proves that
There exists a regular homotopy that turns a sphere inside-out.

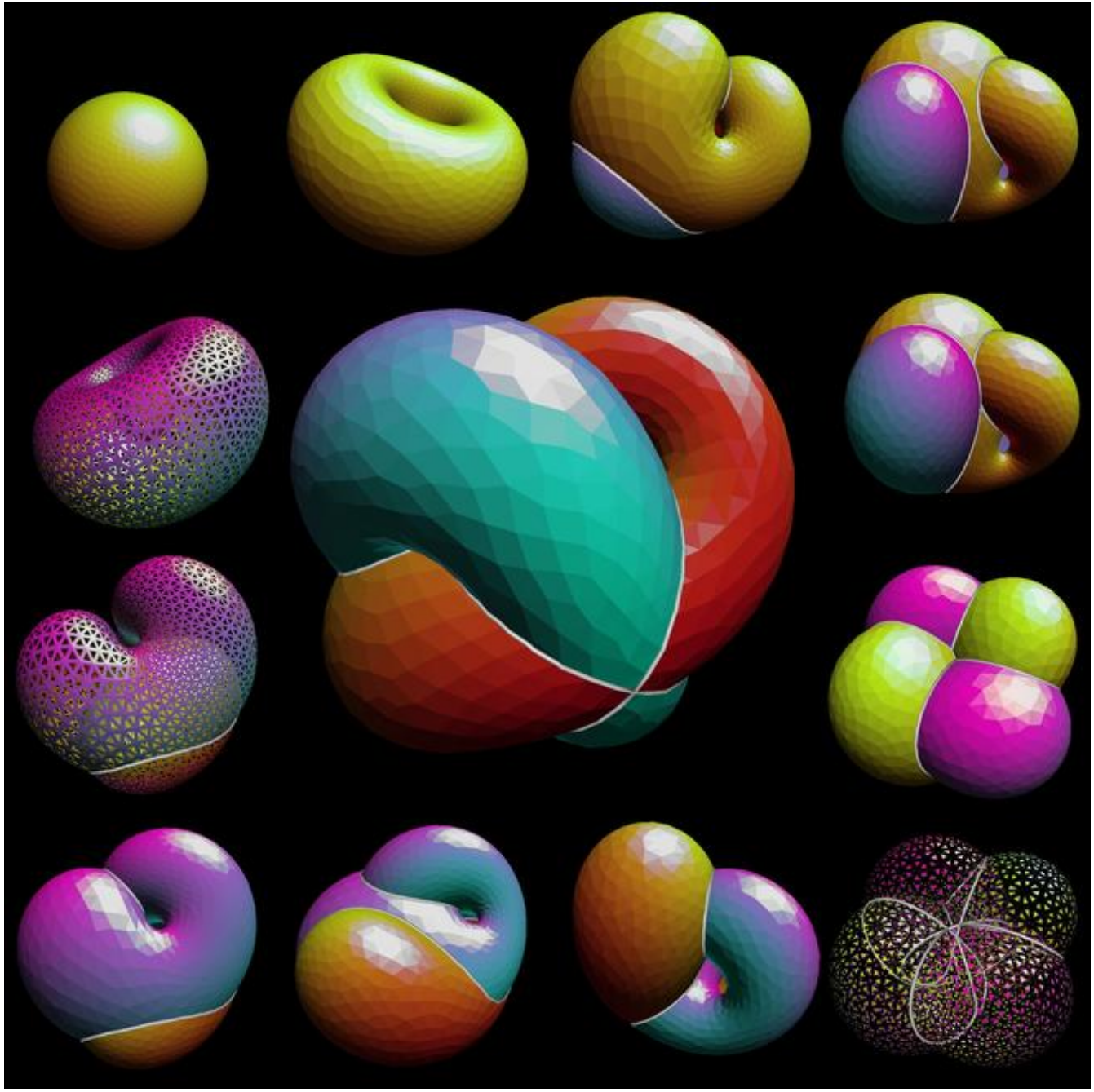
"An unlikely story!" Raoul Bott asks for an explicit example.
1st decade(cartoon drawings):
Arnold Shapiro 1960, Tony Phillips 1966.
2nd decade(computer animations):
Bernard Morin 1967, Nelson Max 1977.
3rd decade (tobacco pouch eversions):
A Topological Picturebook 1987.
4th decade (real-time interactive computer animation):
François Apéry 1992, Richard Denner 1992
Outside In 1995, *Minimax* 1996, *Optiverse* 1998.

Morin Tableau n=2



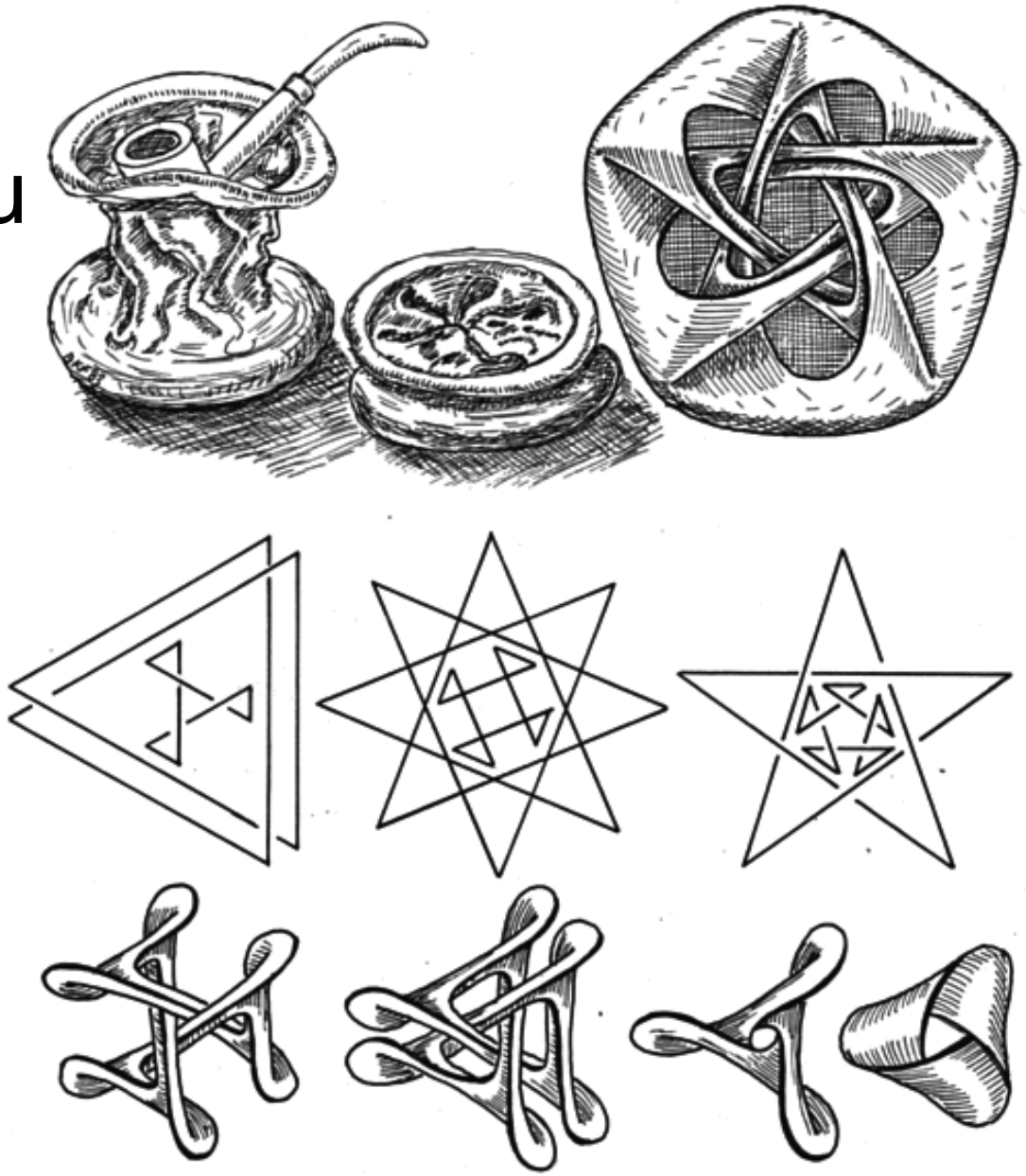
from George K. Francis, “A Topological Picturebook”, Springer-Verlag, 1987

Morin Montage n=2



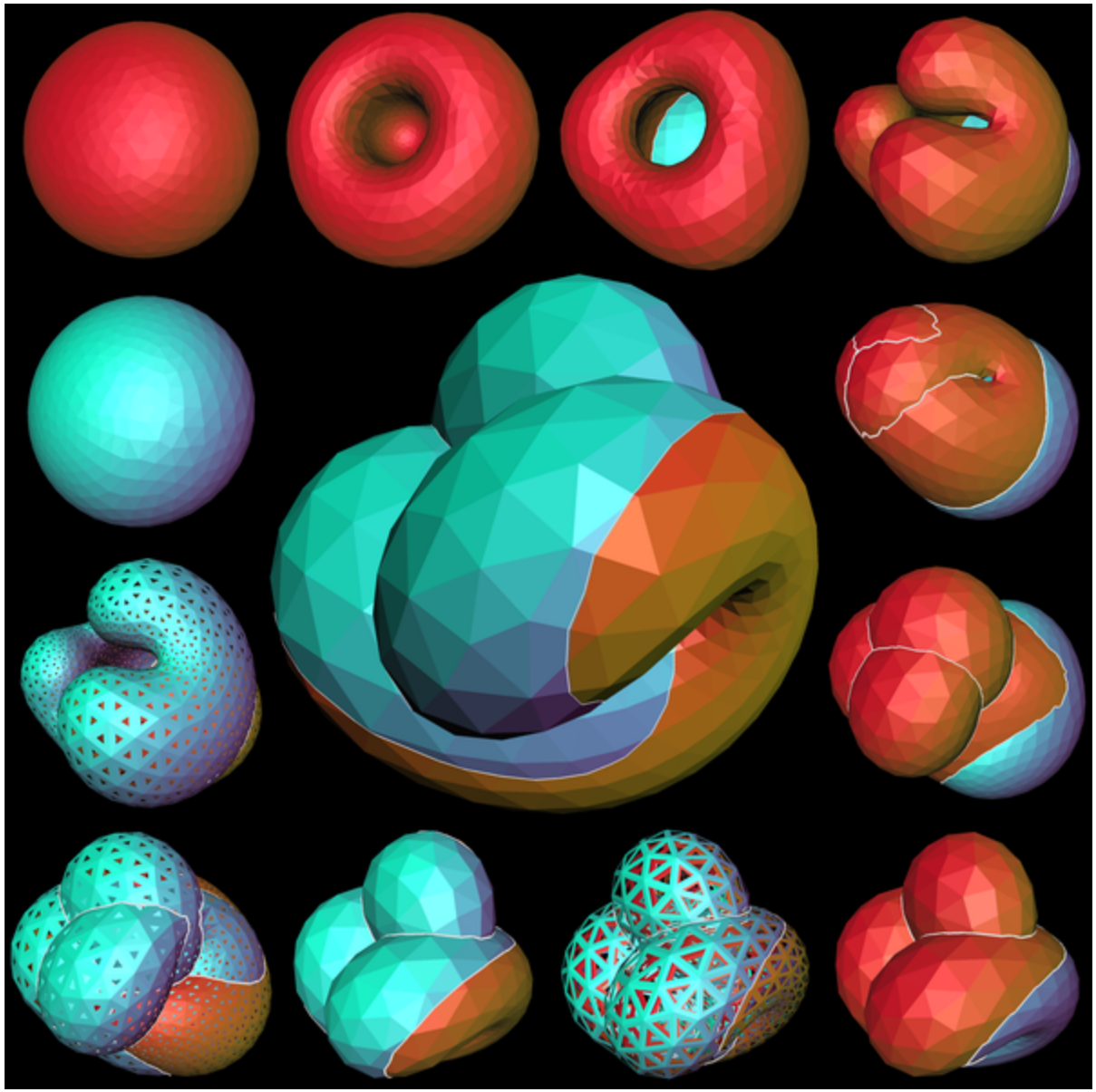
J. Sullivan, G. Francis, and S. Levy, “The Optiverse”, Math & NCSA, U. Illinois, © 1998

Boy Tableau n=3



from George K. Francis, “A Topological Picturebook”, Springer-Verlag, 1987

Boy Montage n=3



J. M. Sullivan, G. Francis, and S. Levy, “The Optiverse”, Math & NCSA, U. Illinois, © 1998

Morin Eversion

1967

2000

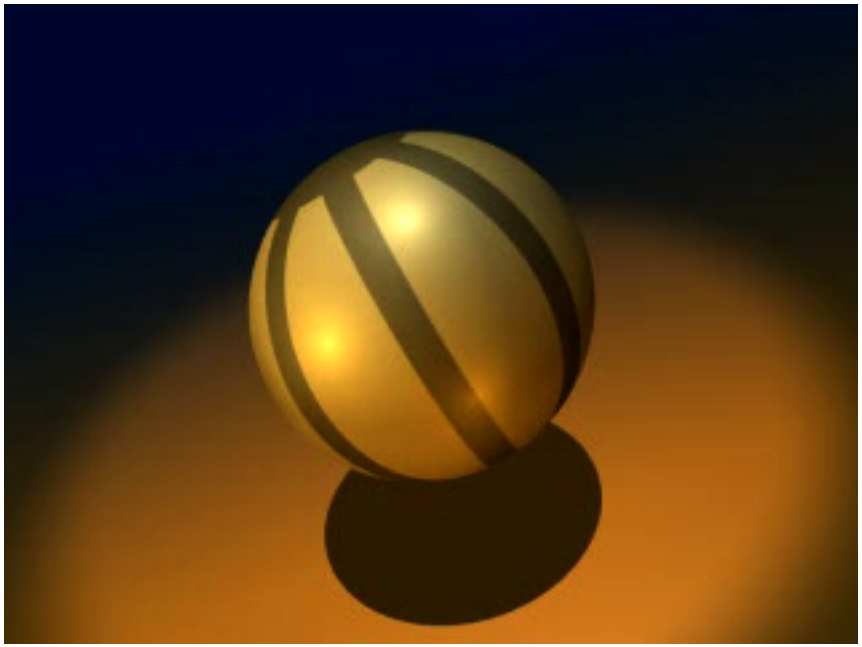
Bernard Morin at Maubeuge, France, 20 September 2000




Bernard Morin

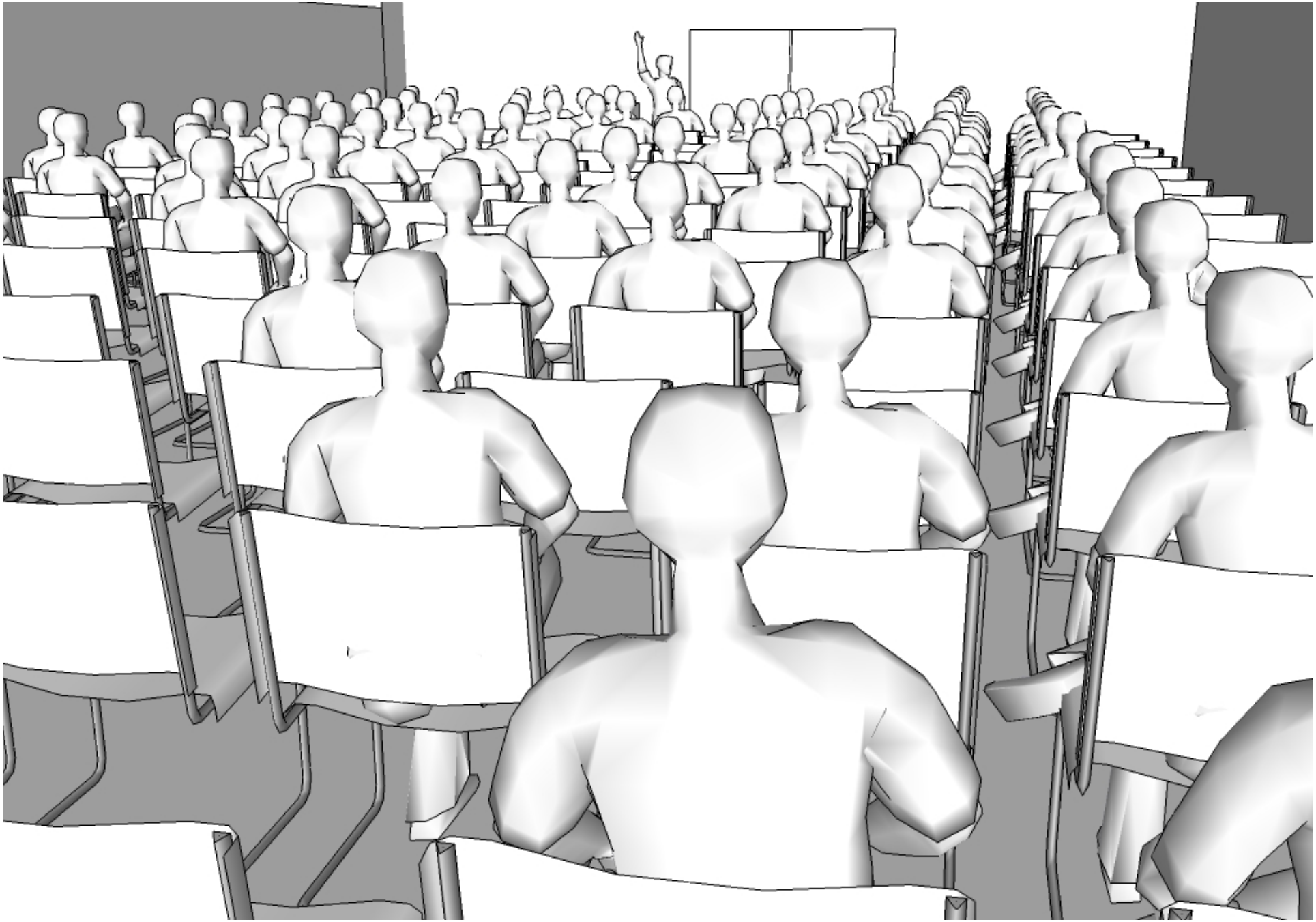
looking at
Stuart Dickson's stereolith models of
John Sullivan's *Minimax Eversion,*
Maubeuge, France, September, 2000

Dickson's Stereoliths

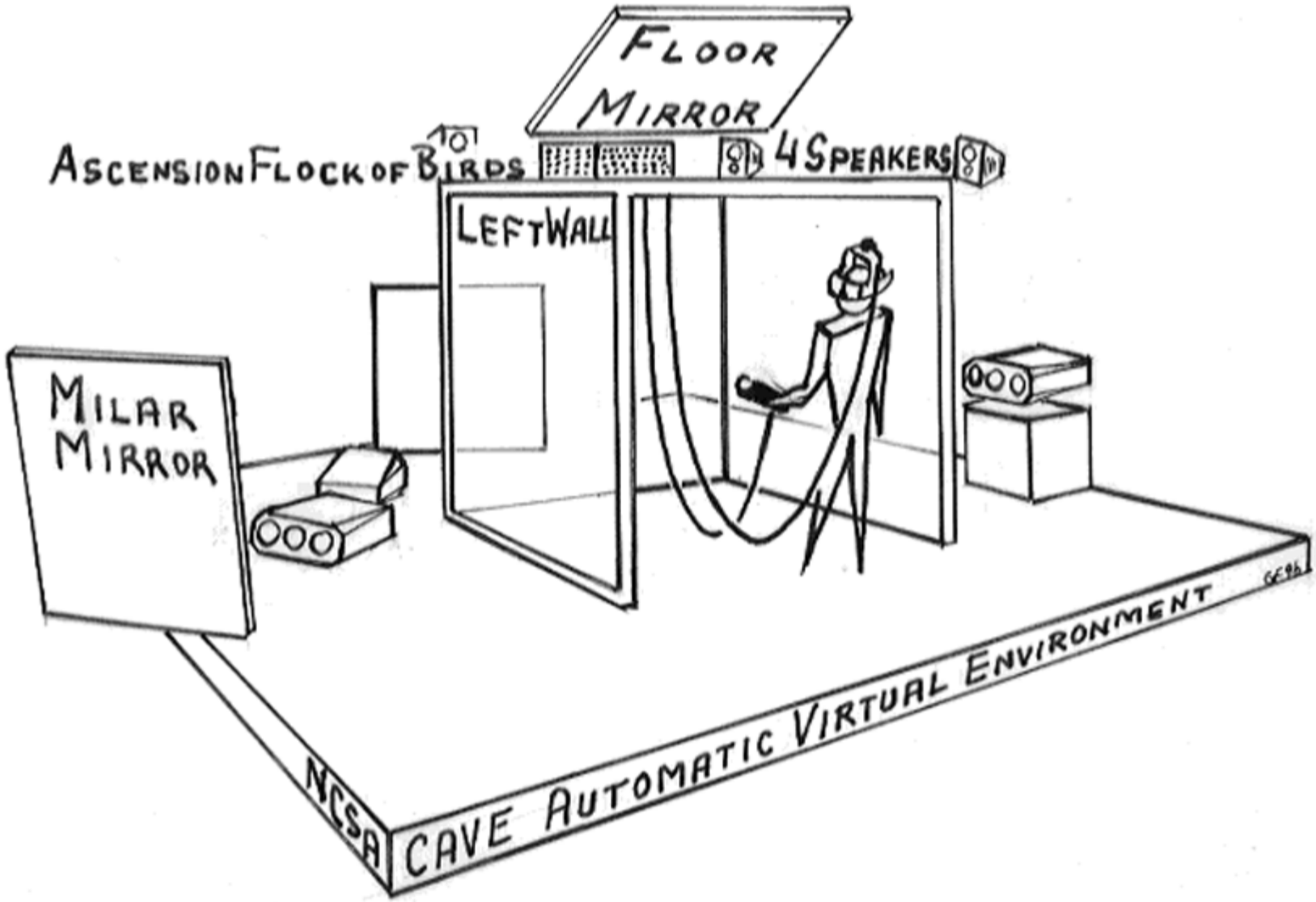


Virtual Reality

Wow, just wait until tomorrow …….



Cnu-atlanta-small-lecture.jpg (Google)


FLOOR
MIRROR
ASCENSION FLOCK OF BIRDS
4 SPEAKERS
LEFT WALL
MILAR
MIRROR
NCSA CAVE AUTOMATIC VIRTUAL ENVIRONMENT

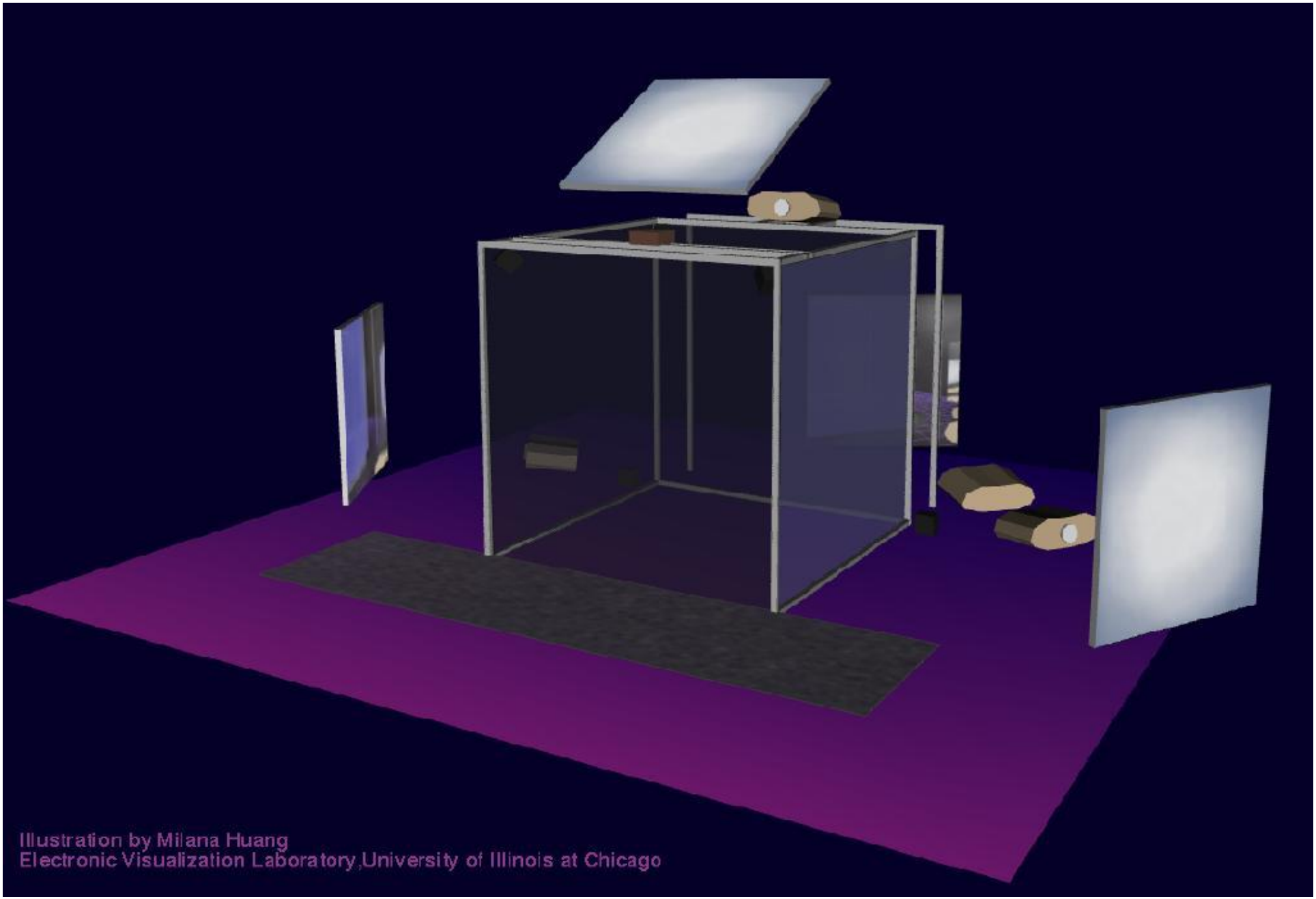
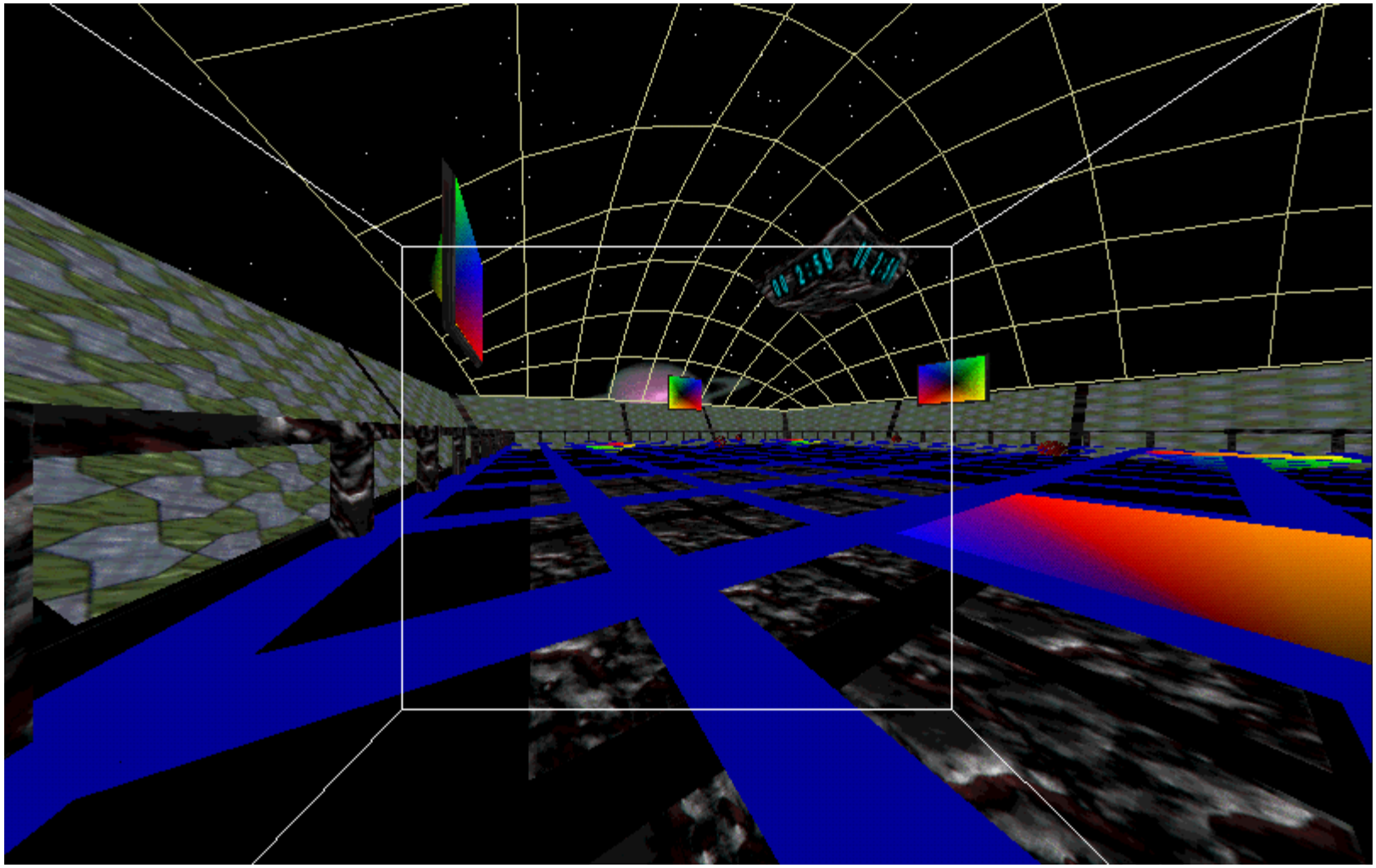


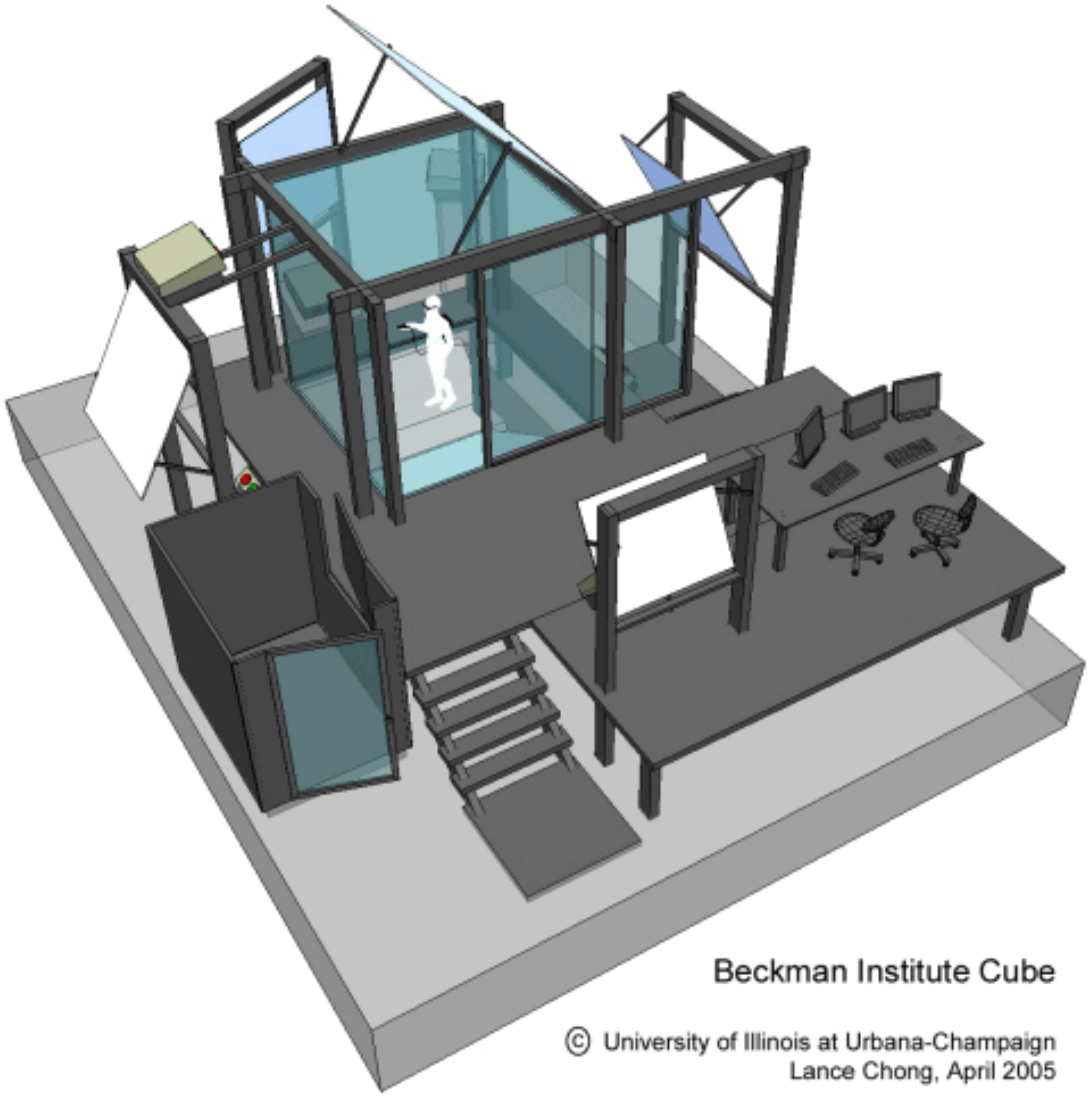
Illustration by Milana Huang
Electronic Visualization Laboratory,University of Illinois at Chicago

Donna Cox and Bob Patterson in the CAVE at the National Center for Supercomputing Applications

Arena

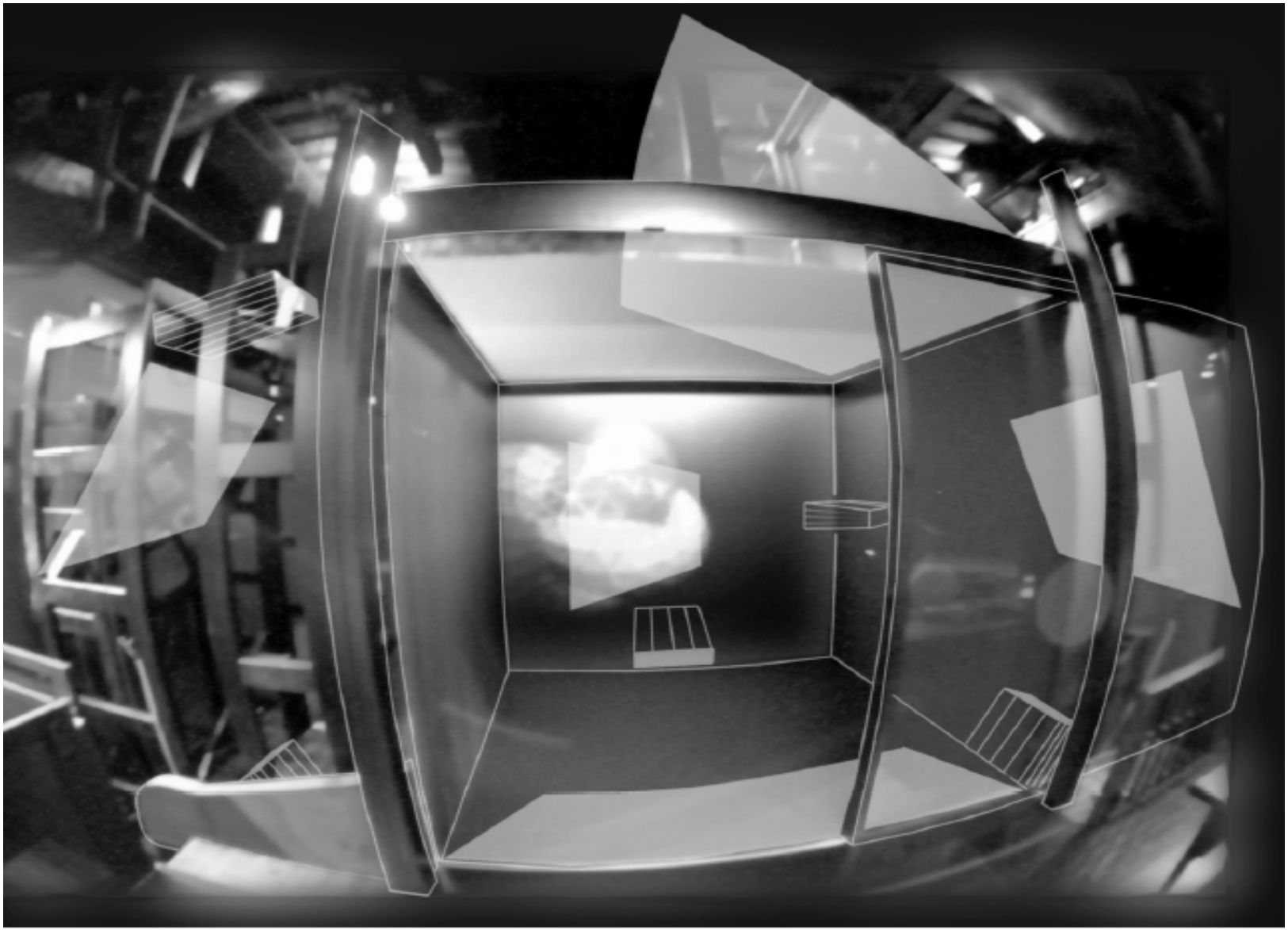


Kevin Vlack, Alexei Bourd “CAVE Gladiator”`NCSA © 1996

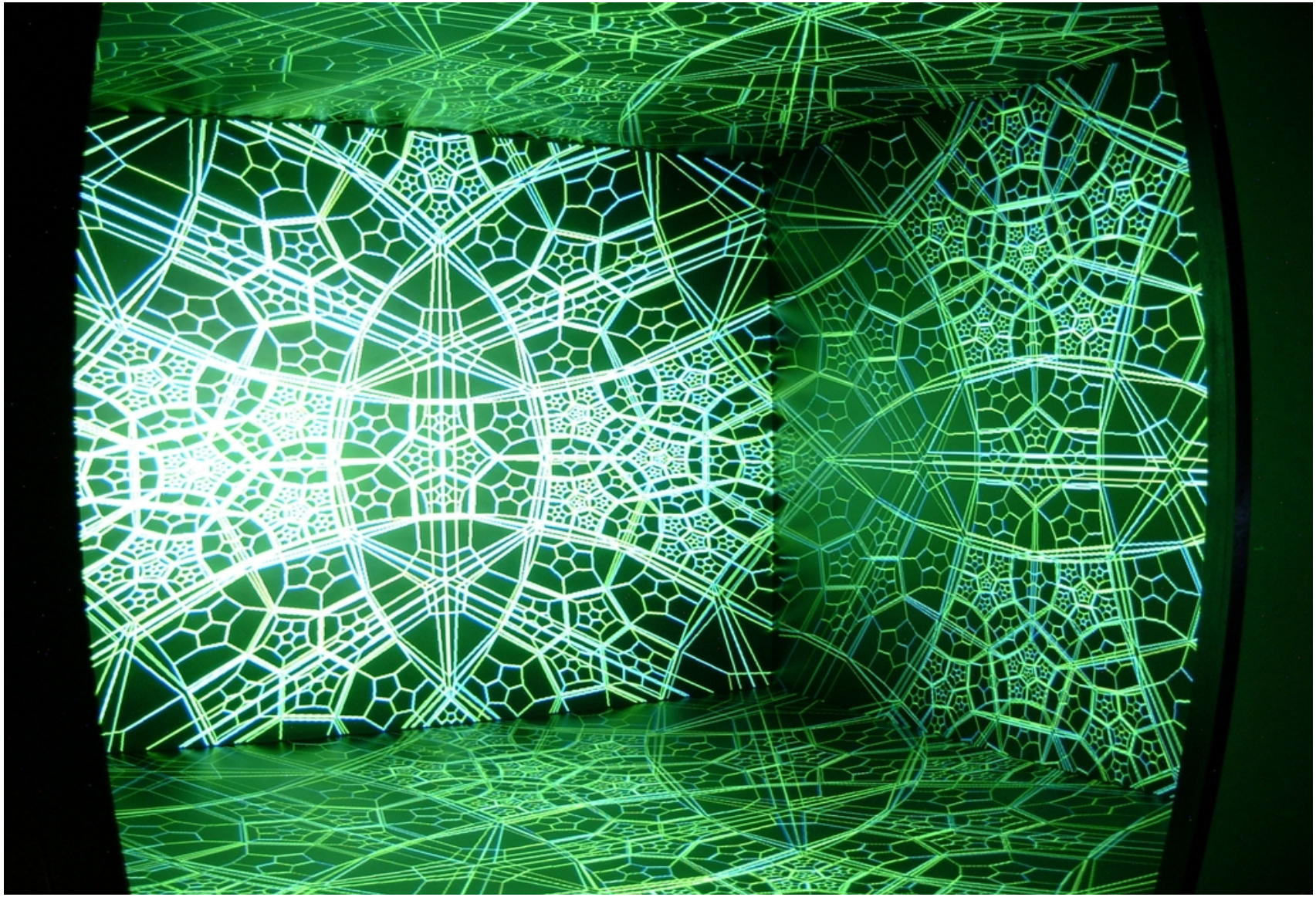


Beckman Institute Cube

Lance Chong, April 2005



“ALICE on the Eightfold Way”, Francis, Goudeseune, Kaczmarski, Schaeffer, Sullivan, 2002



“ALICE on the Eightfold Way”, Francis, Goudeseune, Kaczmarski, Schaeffer, Sullivan, 2002

COAST
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Center for Arts Sciences and Technology at the Danish Technical University
Erik Reitzel - engineer
RCM Precision - fabrication
Poul Ib Hendriksen - photos

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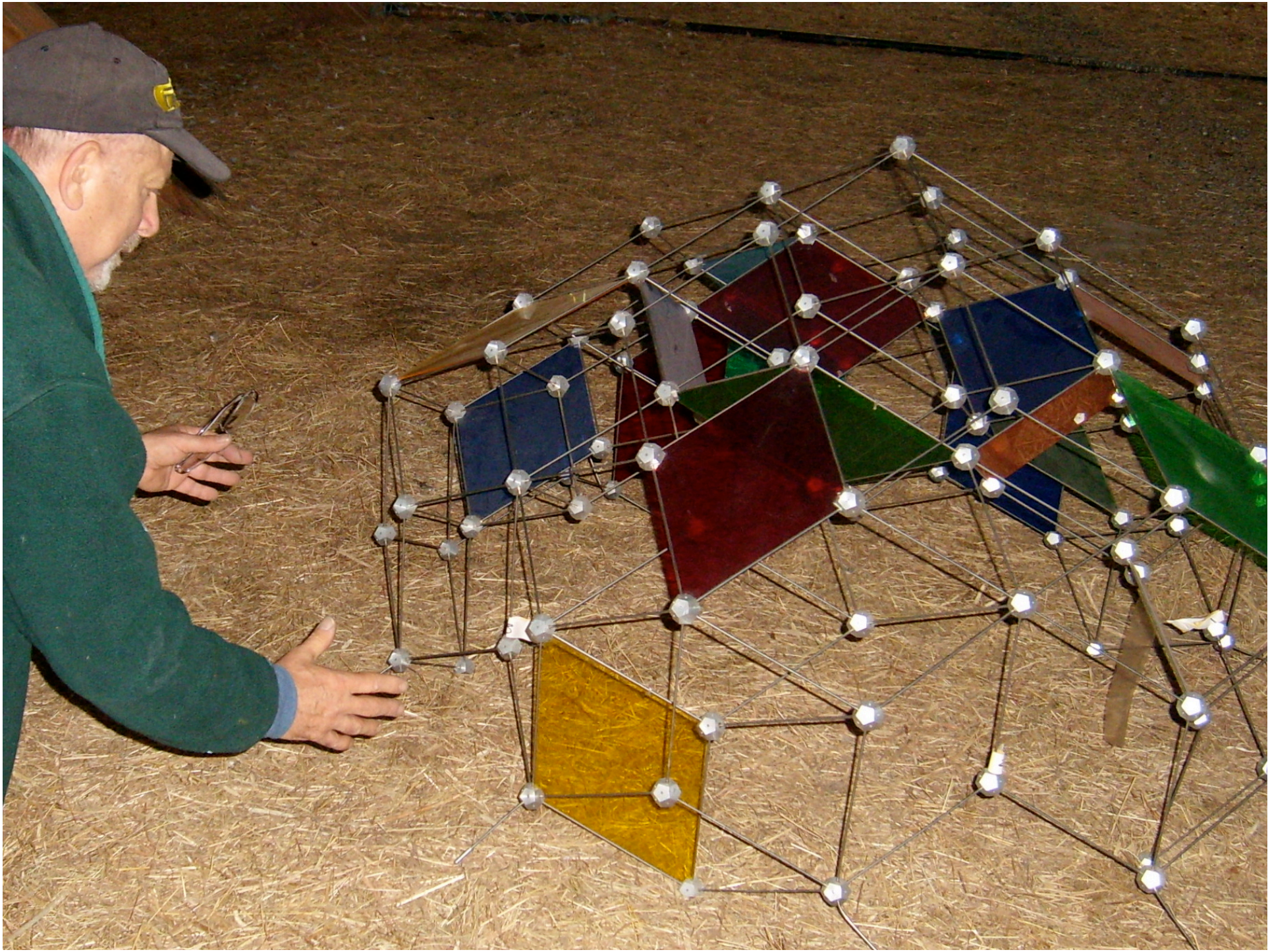
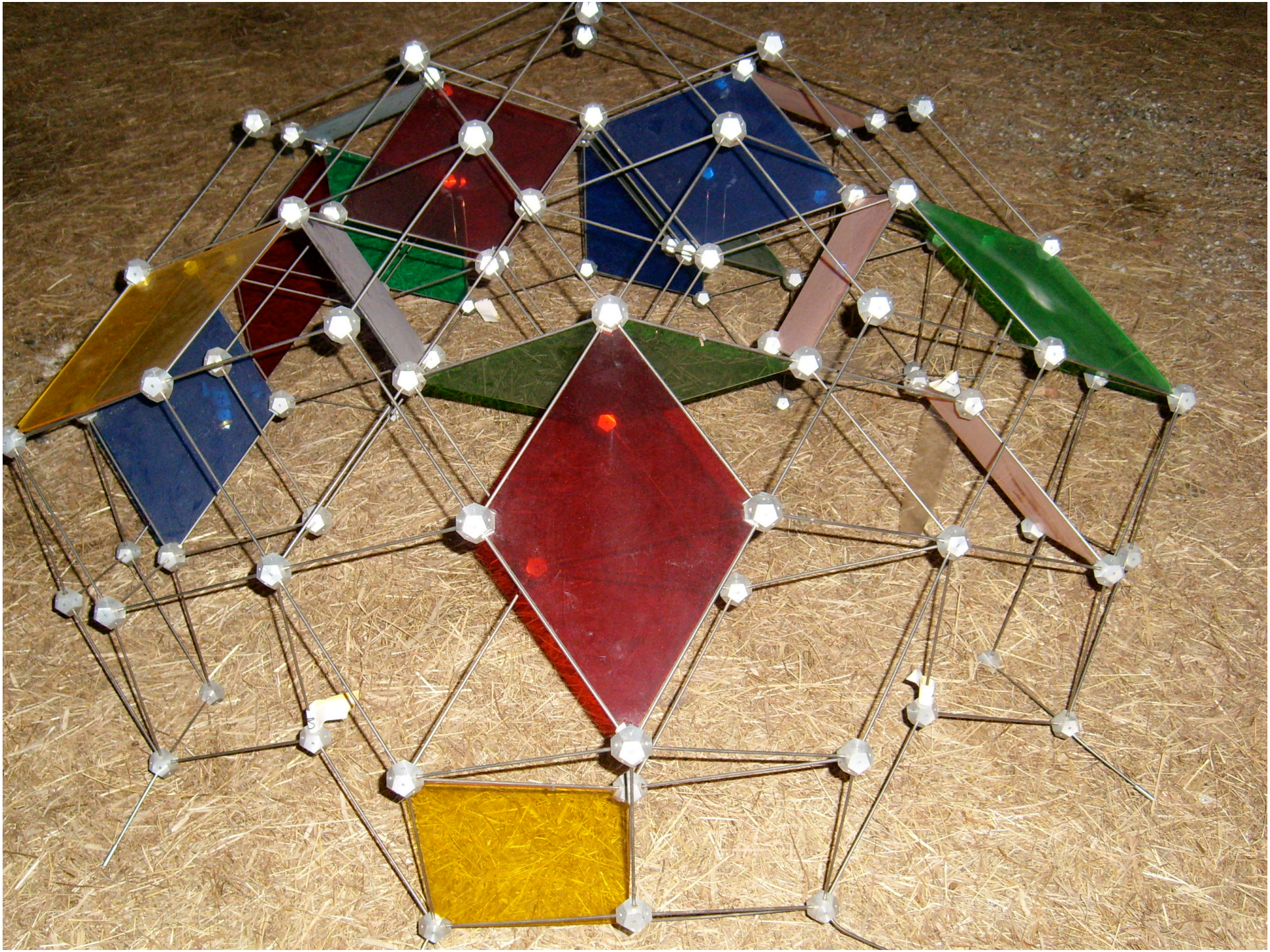
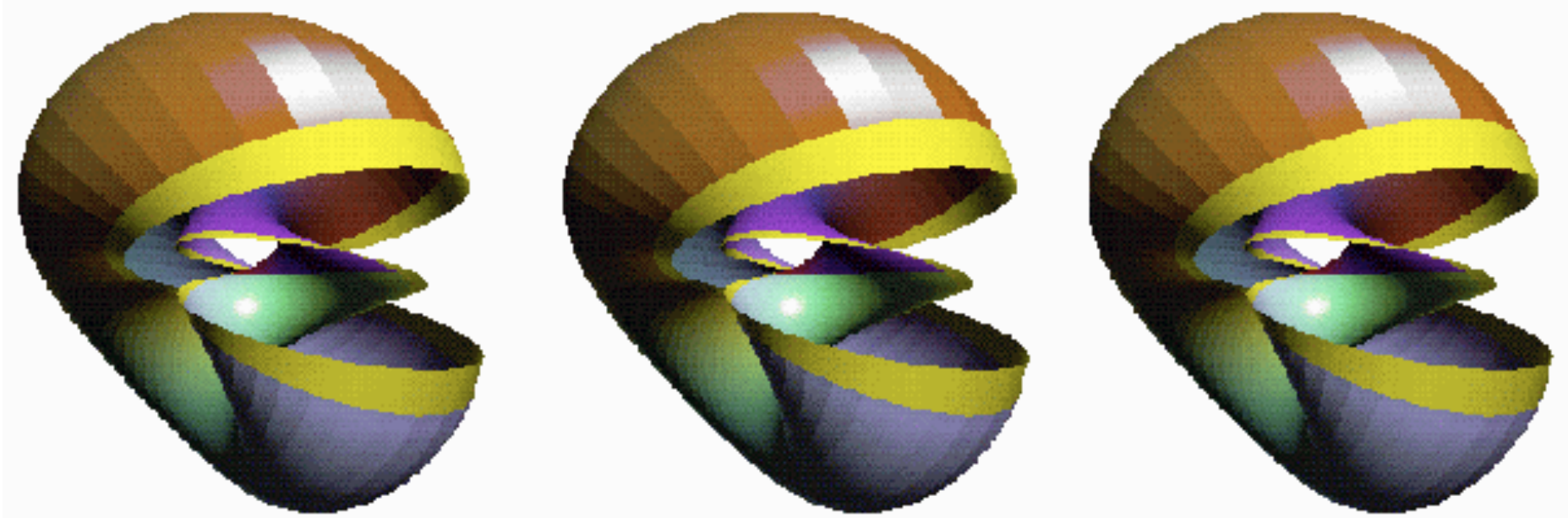




Quasicrystals.py

MA595 Winter 2010 Mini-Course in Geometry: COMPUTER GRAPHICS & GEOMETRICAL VISUALIZATION



Lawson's Snails. Crossed stereo left two, straight stereo right two images.

Professor George Francis
3MWF 102 Altgeld Hall 19jan10-12mar10, 2 credit hours.
Text: Francis, A Topological Picturebook, Springer Paper Back, 2006.

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1993 George Francis, Lou Kauffman, Dan Sandin